DOCUMENT RESUME

ED 050 773   LI 002 835

AUTHOR   Resnikoff, H. L.; Dolby, J. L.
TITLE   Access: A Study of Information Storage and Retrieval
                with Emphasis on Library Information Systems.
                Interim Report.
INSTITUTION   R and D Consultants Co., Los Altos, Calif.
SPONS AGENCY   Office of Education (DHEW), Washington, D.C. Bureau
                of Research.
BUREAU NO   BR-8-0548
PUB DATE   21 May 71
CONTRACT   OEC-0-9-140548-2791(095)
NOTE   225p.

EDRS PRICE   EDRS Price MF-$0.65 HC-$9.87
DESCRIPTORS   *Archives, Books, Indexes (Locaters), *Information
                Retrieval, *Information Storage, Information
                Systems, *Library Collections, *Library Materials

ABSTRACT
                Chapter I: "Introduction and Summary of Results,"
stresses the view that the problem of insufficient access is
primarily a problem of the great size of the archives to which access
is desired. Chapter II: "Levels of Information Storage and Access,"
is directed toward the problems of library archives and in this
context it is access to the content of books and collections of books
that is of immediate concern. Chapter III: "Mathematics of
Information Distributions," is devoted to the mathematical study of
some of the distributions that arise naturally in the study of
information systems. Chapter IV: "The Structure of the Back-of-the
Book Indexes," is a study of indexes to books in order to determine
what structure, if any, they possess. Chapter V: "Algorithmetic Text
Indexing," is also exclusively concerned with back-of-the book
indexes. Chapter VI: "Amalgamative Access Mechanisms," looks at the
problem of discovering possible methods for accessing books. Examples
of indexes are appended and the tables included are listed. (NH)

INTERIM REPORT

PROJECT NO. 8-0548

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ED050773

ACCESS

A STUDY OF INFORMATION STORAGE AND

RETRIEVAL WITH EMPHASIS ON LIBRARY

INFORMATION SYSTEMS.

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21 May 1971

U. S. DEPARTMENT OF

HEALTH, EDUCATION, AND WELFARE

Office of Education
Bureau of Research

I 002 835

ACKNOWLEDGEMENT

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CHAPTER I

INTRODUCTION

AND

SUMMARY OF RESULTS

INTRODUCTION AND

SUMMARY OF RESULTS

This monograph describes work performed by R & D
Consultants Company during the first twenty-six months
of contract #OEC-0-9-140548-2791(095) with the Office
of Education of the Department of Health, Education,
and Welfare. The contract is titled "A computer-aided
study of access management and collection management
in libraries"; its principal objectives are the
development of a model for information access and
storage systems, and the study of the structure of
existing access systems with the intent of augmenting
them in significantly useful ways by means of auto-
mated processing of machinable data bases.

The concern which underlies this and many other projects
is that the rapidly growing body of information stored
in library archives is overwhelming the traditional
means of obtaining access to it in a reliable, timely,
and comprehensive manner.

In fact, general archival collections have been growing
in an essentially exponential manner for more than three
hundred years, and perhaps for much longer. Figure 1.1,
drawn on semilogarithmic graph paper, illustrates this
phenomenon for serials noted in the Union List through
1930; this collection has been doubling in size every
thirty years. If the trend line is extended back in
time, it suggests the publication of a "first" printed
serial about 1435, which agrees remarkably well with
the invention of printing circa 1440-1456. Although the
weight of this evidence is insufficient to convincingly
show that serial growth has in fact been exponential
since that time, it does support the contention that
the exponential growth of archives is a fundamentally
long-term property, undoubtedly secondary only to
economic and population growth and determined by them.
Consequently, it must be anticipated that archival
growth will, for the forseeable future, continue to be
exponential apart perhaps from fluctuations of minor
duration because there does not yet appear to be a signif-
icant slackening in either population or long term
economic growth for the world as a whole.

Figure 1.1

NUMBER OF SERIAL PUBLICATIONS
WITH TREND LINE

(UNION LIST OF SERIALS IN LIBRARIES
OF THE UNITED STATES & CANADA ,
3RD EDITION)

Some insight into why this "information explosion" has received particular emphasis in recent years can be gained from a study of Figure 1.2, adapted from De Solla Price (1) which shows that in addition to the exponential growth of the number of scientific journals since the second half of the seventeenth century, the number of scientific abstract journals has also been growing exponentially, and at the same rate, since their introduction in about 1825. The abstract journals provide access to the larger body of primary scientific journals; the figure shows that the need for this secondary form of publication apparently appeared when the number of scientific journals reached 300. The number of abstract journals reached 300 by 1950, making it as difficult to access the abstract journals as it had been to access the primary archive in 1825. This suggests that one of the reasons for the current serious concern about problems of information storage and retrieval is that it is once again necessary to invent an appropriate form of (tertiary) publication which will permit another period of orderly growth of the archive.

If the historical and current trend continues unabated for another fifteen years, there will be about 500,000 different scientific journals in existence, publishing more than 25 million papers each year; similar quantities of information will be spewed forth by other fields of endeavour. It is clearly not the problem of storing this information that makes the prospect of such prolific productivity terrifying; current microform techniques are already sufficient to reduce the physical storage requirements to much less than that presently required to store the current production of journals published in conventional form. Moreover, standardization of microform stores make it possible to implement physical retrieval systems that are faster and cheaper than present typical library storage techniques. Nor is the prospect of having to read all of the published material the significant problem. No scientist since 1800 has had the time to read "all" of the papers published even had he the inclination to do so; the situation is the same in most other fields. The inevitable fact that the fraction of published papers read by an individual is going to drop a few more orders of magnitude is hardly consequential.

The problem posed by the explosion of information is only overwhelming when the difficulty of finding a particular fact or result in the vast sea of information is considered. It is this problem of access to which the work reported here is addressed.

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Figure 1.2

Exponential Growth of Scientific
Journals and Abstract Journals

Number of Scientific Periodicals (Data from D. J.
de Solla Price: *Science since Babylon* [New
Haven, 1961], p. 97).

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Chapter II introduces a level structured model for
access systems, which can be briefly described here.
Restricting attention to collections of information ex-
pressed in natural languages, size can be reasonably
measured by the number of characters, including linguistic-
ally necessary interword spaces, contained in the collection.
For naturally occurring informational units such as
the book title, table of contents, book index, book,
and, regarding amalgamated information stores, the
university library card catalog and the university
library itself, the average size of each informational
unit is nearly an integral power of a fixed number K
of characters. The value of K is nearly 30. For
example, $K \sim 30$ is the average length of a book title
measured in characters (as well as the average length
of an index entry and of the subject heading information
on a Library of Congress catalog card); $K^2 = 874$ char-
acters is approximately the average size of a table of
contents; $K^3 = 25,822$ of the average book index, and $K^4 =$
763,203 of the average book. In each case, the average
length is remarkably close in value to the power of K
in question.

If the size of an information collection is expressed
as a power of K, say K^x, then it is convenient to define
the level of the collection as the integer closest to
x. With this convention, the level of a book title,
table of contents, book index, and book is, respectively,
1,2,3, and 4. A university library is of level 8.
It therefore appears that the traditional means for
retrieving information stored in a book are structured
in levels which are equally spaced when measured by
their level, that is, when measured by the logarithm
of their size.

If one information base is an access system for another,
as a book index is for a book, then the order of
access is defined to be the difference between their
levels. In general, the larger the order, the less
expensive is the access system insofar as its construction
and maintenance are concerned, relative of course to
the cost of obtaining and maintaining the accessed data
base; but the smaller the order, the more effective the
access system will be in locating specific information
and accurately reflecting the content of the accessed
archive. For instance, a title list is less expensive
and less informative than a collection of abstracts
(such as Chemical Abstracts) in specifying the content
of journal articles in chemistry; the former is of
order 2, the latter of order 1.

The level structure described above will provide a
valuable management tool for determining, amongst other
things, the reasonable size and cost for a system de-
signed to access a given information base only if the
average size of a class of information bases is typical
of the distribution of sizes in that class. That this
is indeed the case is strongly attested by extensive
data sampling studies presented in Chapter II, including
the analysis of more than 500,000 index entries occurring
in a random sample of books drawn from a medium size
university library. All of the evidence reinforces
the hypothesis that the distribution of size
of information collections belonging to a class is
lognormal; that is, the distribution of the logarithm
of the size of the informational units belonging to
the class is a normal distribution. Each access level
corresponds to a different lognormal distribution. It
turns out that the variance of the occurring distri-
butions are all nearly the same throughout the entire
range from level 1 (titles) to level 8 (university
libraries); this means that the distributions depend
essentially only on their mean and are therefore character-
ized by their level. This justifies the use of the
notion of level as a measure of an access system.

The principal objective of Chapter III is to show that
the lognormal distribution of size of informational
units belonging to a class (e.g., titles, books, libraries)
is a mathematical consequence of certain reasonable
assumptions concerning the "effort" or "cost" of using
an item in an access system if the complete system maxi-
mizes the output of information per unit effort expended.
Our argument is a minor extension of Mandelbrot's
derivation of the generalized Zipf-Bradford distribution;
cp. Refs. (2), (3). The remainder of the chapter describes
general mathematical properties of lognormal distributions
with emphasis on the most convenient but nevertheless
laborious and not entirely satisfactory technique for
fitting lognormal functions to sample data; a number of
worked examples which are of independent interest
are included.

Unfortunately we do not know a theoretical argument that
will produce the equispacing of the level structure
of the means of the lognormal distributions associated
with an access system; this aspect of the access model
rests entirely on observational evidence.

The book is still the most natural informational unit
for those concerned with library matters. According to
the access model, there are exactly four orders of access
associated with collections of information of this size,

and, as we have already remarked, there is a tradi-
tional access system operating at each of these levels:
the index is of order 1, the table of contents of
order 2, the title of order 3, and finally, the
Library of Congress letter class, which partitions the
entire span of written human knowledge into 21 grand
categories, is of order 4.

Although these access levels are, in accordance with
the prescription of the model, the only ones possible,
there are of course many different types of access
systems which can function at each of these levels
in addition to those just named. For example, a
nine page review of a 277 page book provides typical order
1 access for the book of average size. Order 1 access
systems most accurately reflect the content of the
information collection they access and can moreover
form the subsidiary information base from which access
systems of higher order (i.e., lower level) can be
constructed. Because this procedure obviously cannot
be reversed--a low order access system can never be
constructed from one of higher order--order 1 access
systems deserve special study.

Of the traditional order 1 access systems, the book
index is the most amenable to extensive statistical
analysis, both because it is found in close proximity
to the book text to which it refers (which is generally
not the case for book reviews) and because it is naturally
composed of a large number of homologous small entities
which are suitably arranged for analytical study.

We have investigated three major collections of book
indexes. The first contains more than 100 books drawn
from the present authors' libraries; although this sample
exhibits some variation in subject matter, science and
more particularly history, mathematics, and physics are
heavily weighted. The second sample consists of 80
current books in statistics and probability theory
and comprise what can be thought of as a specialists
hand library; it undoubtedly accurately reflects the
nature of indexes to books in these fields. All index
entries in this sample were committed to machine readable
form to permit ready reorganization and analysis of the
collection of index entries, of which there were 31,232.

Study of these collections was instrumental in guiding
us to the formulation of the access model presented in
Chapter II, but their limitations--principally their
restriction to few subject areas and the undoubtedly
biassed method of their selection, but also their rela-
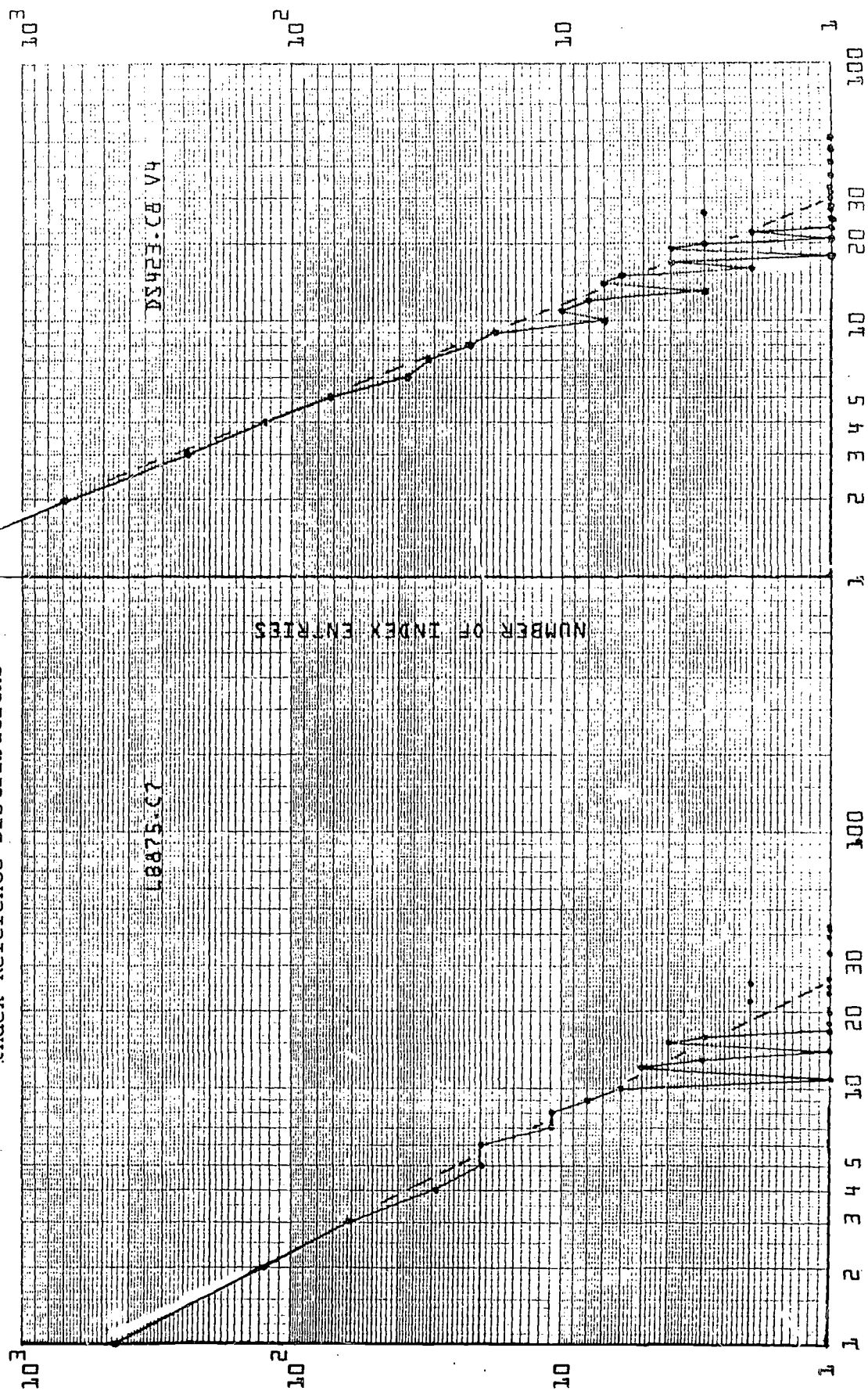tively small size--clearly indicated the desirability

of carefully selecting a random sample of books and
their indexes from a broadly representative archive.
The third index sample consists of such a random selection
of 706 indexes from the Fondren Library at Rice University.
For each book in this sample, copies of the shelf
list catalog card, title page, table of contents, and
index were made. From this information it is possible
to determine the size of the book (in pages, which can
then be approximately adjusted to equivalent number
of characters) and the precise number of characters in
the title, table of contents, and index, which are the
three significant traditional book access systems that
are normally packaged with the book itself.

Chapter IV describes the structure and properties of
traditional back of the book indexes based on a study
of these three samples. There are three main conclu-
sions: first, the average number of index entries per
index is determined; the result is 836, with relatively
little variation throughout the different Library of
Congress letter classes. Second, it is shown that
the distribution of the number of books as a function
of the number of entries in their index is lognormal,
providing further support for the access model derived in
Chapter II. The remainder of Chapter IV is devoted to
a study of the distribution of the number of text
references per index term in a given book. The under-
lying idea is an outgrowth of the simple observation
that those index entries that refer to only one text
page cannot typify the general content of the book,
whereas an entry that refers the reader to 40 or 50
text pages is truly of little specific utility to the
reader except insofar as it points out one general topic
of the book. It is therefore conceivable that some
subset of the index functions as a collection of "key
words", specifying the semantic content of the work and
serving little further purpose. Were it possible to
separate this subset from the other more numerous index
entries, the way would be clear for automatic descriptor
determination based on a machine readable index; moreover,
if the process of constructing the index itself could
be automated, iteration of these processes would lead
to the descriptors as well and quite possibly to a
successful method for man-machine interactive content
classification.

Figure 1.3 exhibits the page reference distribution
for two books; LB875.C7 was published in 1922, is titled
Two Views of Education, and contains 775 index entries,
whereas DS423.C85 v4 is the fourth volume of The Cultural
Heritage of India, published in the interval 1953-58, and

Figure 1.3

Index Reference Distributions

containing 4906 index entries. It is clear from the figure,
which is drawn on full logarithmic graph paper, that
except for the quite small numbers of entries referring
to very many pages both distributions are linear on
the graph paper and hence the number of index entries
is a power function of the number of page references.
From the theoretical considerations in Chapter III
one is led to suspect that these graphs ought perhaps to
represent lognormal functions, which appear on logarithmic
graph paper as parabolas. As we show in the third
chapter, the power function, represented by a straight
line, is a degenerate form of the lognormal representing
parabola. Other book indexes, as for instance that of
The 1969 World Almanac, illustrated at the left half of
Figure 1.4, do indeed flaunt a tell-tale curvature and
can be accurately fitted by a parabola. This is the
third major result of Chapter IV: the page reference
distribution of index terms is, generally, a lognormal
function which may degenerate into a power function.

There are about 6600 index entries in The 1969 World
Almanac. This book and others like it are more thoroughly
and densely indexed than most, but let us for the moment
treat this index as an order 1 access system for its
text, as usual, but simultaneously consider it as an
information base requiring access systems. Then selection
of that 1/30 of the index which refers to the largest
number of page locations will produce an order 2 access
system for the original text, which will be of the size
of a table of contents. Repeating this operation leads
to the selection of the subset of the index which is about
$1/(30)^2$ = 1/900 the size of the index and approximately
the size of a title. This process produces about 8
index entries, substantially larger than a title because
of the peculiarities of almanacs. Figure 1.4 shows the
four most "popular" index entries; in approximately
the space of the title they provide an order 3 precis
of the book's content which is a not unuseful alternate
to the title itself.

The distribution for Nader's Unsafe at any Speed is shown
in the same figure; the three most popular index entries
again provide a cogently descriptive view of the book's
content which is in fact not provided at all by the title.

Chapter IV pursues the study of the effectiveness of the
popular index entries as content descriptors through
the analysis of a uniform subsample of the Index Sample;
for this subsample those index entries which refer to
large numbers of text locations have been explicitly
listed. The subsample is included as Appendix I.

Figure 1.4

High Frequency Index References

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The problem of automatically indexing documents and
books has intrigued computer buffs for years. Numerous
programs are now available and many learned research
papers have been written describing them, and how modest
are their demands on the machines that implement them,
and how effective they are in satisfying rather impre-
cisely stated hypothetical requirements of potential
users. But as far as we have been able to learn, no
commercial or professional publishing house uses machines
to index books or papers. The reason that this is so
consists of a complex of subreasons not all of which
have to do with the adequacy of machine methods, but
it is certainly true that the general complexity,
inflexibility, and simple inadequacy of these programs
have acted as strong deterents to their use. Although
the problem is hardly a trivial one, we think that one of
the most significant factors hindering the development
of indexing algorithms that will rival and surpass
human performance is that no one has ever attempted
to assess precisely what properties human produced
indexes actually have as opposed to what indexers and
students of indexing believe ought to be the properties
of indexes. The availability of the Fondren Index
Sample has made it possible to assess human performance
in this area, and to set standards for the performance
of machine methods of indexing which are objective, and,
insofar as they refer to the structural statistics
of indexes rather than their semantic content, also
measurable. Elucidation of these common structural
characteristics of human indexes have in turn suggested
some new approaches to the problem of machine indexing.
Chapter V is devoted to one such new method. Figure 1.5
illustrates the text location and page location reference
distributions for the algorithmically produced index
to Computerized Library Catalogs: Their Growth, Cost
and Utility. Based on our study of the Fondren Index
Sample we can assert that this algorithmic index is
the "right size"; moreover, it is evident from the
figure that the reference distributions agree well
with typical distributions associated with human indexes.
The details of the algorithm as well as of the index
referred to by the figure are the subject of the fifth
chapter.

Combining these results with those of the previous
chapter leads to a new method for obtaining keyword
descriptors, which is discussed in the context of the
particular algorithmic index exhibited. Indeed, this
index consists of 340 entries; an order 1 access system
acting on this index should select about 12 entries
which would provide an "abstract" of the content of the
index. There are 9 entries referring to at least ten
page locations, but 14 referring to at least nine. Table
1.1 lists these 14 abstract entries together with the
number of pages to which each refers.
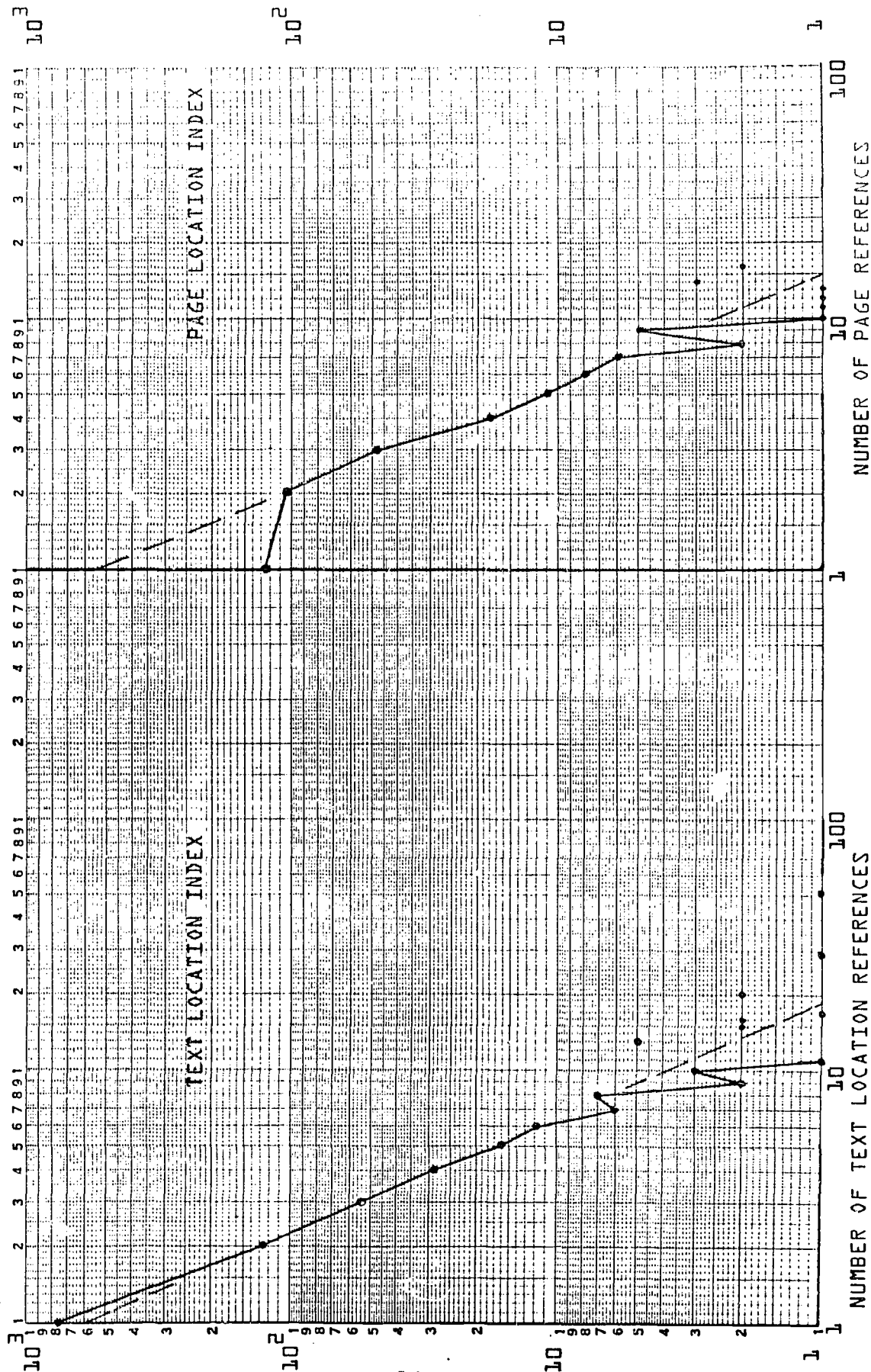
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Figure 1.5

Text and Page Location Distributions

PAGE LOCATION INDEX

TEXT LOCATION INDEX

NUMBER OF PAGE REFERENCES

NUMBER OF TEXT LOCATION REFERENCES

ALGORITHMIC INDEX

▽COMPUTERIZED LIBRARY CATALOGS: THEIR GROWTH, COST, AND UTILITY▽

Table 1.1
ABSTRACT INDEX ENTRIES
FROM
"COMPUTERIZED LIBRARY CATALOGS:..."

No. of Page References	Index Entry
16	LC
16	GNP
14	growth rate
14	library catalog
14	machine-readable form
13	Library of Congress
12	gross national product
11	university library
10	exponential growth
9	bibliographic record
9	Fondren, see Rice University
9	Sample, see Fondren Sample, Rice University
9	shelf list
9	Stanford

In this case the abstract entries provide an accurate
capsule view of the problems studied in that book as
well as a list of the principal sources of information
upon which it bases its arguments.

Although there are only four levels available for
accessing the text of books and each of these is already
served by a traditional access mechanism, there remain
many possibilities for repackaging access information
in order to serve needs that cannot be met by traditional
means. Many of these are amalgamative in the sense that
they combine access information associated with numerous
comparable unit information stores in a reorganized
manner that permits ready selection of the units that
are likely to contain specific matter desired by the
user. All document information retrieval systems
operate in an amalgamative manner, as does the library
catalog. Most low order (high level) amalgamative
access systems currently in use organize the access
information in a sequential fashion based first on
date of publication and secondarily according to some
scheme of content classification. This is the procedure
used to organize professional society abstracting journals
(which provide order 1 access); its success depends
entirely on the accuracy and excellence of the content
classification system and the classifiers who implement
it. Such systems, which represent professional consensus
concerning significant categorical classifications,
are in general partly obsolete, especially in rapidly
growing fields such as chemistry where the quantity
of published material may double in as few as eleven
years. Moreover, although the bulk of the classified
material remains stable as the classification system
expands and is refined, some fraction of the archival
materials, which is likely to include the most innovative
work, should be reclassified to account for changes in
classification categories and procedures, but due to
economic constraints, it never is. This difficulty
suggests that it may be desirable to investigate amal-
gamative access mechanisms which do not depend on external
classification structures which are inherently slow to
accommodate themselves to change but rather rely on the
text terms and systems based on the processing of numerous
homologous small items such as index entries, as opposed
to text abstracts, which are of special interest because
they are less subject to global grammatical constraints
and therefore admit a greater variety of potentially
useful orderings.

Chapter VI studies two new types of amalgamative access
systems. The first consists of the combined indexes to a

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collection of books, here illustrated by the combined
indexes to 80 books in statistics and probability theory,
already mentioned above in another context. The second
is more unusual. We have applied a version of the
algorithmic indexing procedure described in Chapter V
to two samples of 50 abstracts drawn, respectively,
from the Annals of Mathematical Statistics and the
Journal of Cancer Research; the results are exhibited
and analyzed.

Appendix I displays the order 1 abstract entries (but
in some cases involving exceptionally large indexes,
only the order 2 abstract entries) from a uniform sub-
sample of the Fondren Index Sample.

Appendix II displays the distribution of the number of
index entries as a function of the number of distinct text
pages to which each entry refers for the same subsample
of the Fondren Index Sample used for Appendix I.
These distributions confirm the assertion that the dis-
tribution is essentially a power function.

Appendices III and IV are automatically constructed
amalgamative indexes to 50 abstracts of papers in
statistics and in cancer research respectively. The
algorithm and all internal dictionary-like stores
used by it is the same for both data sources.

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CHAPTER II

LEVELS OF

INFORMATION STORAGE AND ACCESS

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In the previous chapter we have stressed the view that
the problem of insufficient access is primarily a
problem of the great size of the archive to which access
is desired. This study is directed toward problems of
library archives and in this context it is access to
the content of books and collections of books that is of
immediate concern although libraries are increasingly
becoming archival depositories of other types of
information bearing records.

There are technical reasons that make it desirable to
restrict attention--at least in a preliminary study
such as the present one--to the monograph collection;
we will have some useful remarks to make about serials
and can also exhibit data supporting the extension of
the model that will be proposed to describe the serial
collection.

The book is a natural halfway house in the hierarchy
of means for storing written information in libraries.
Within the book are usually to be found certain standard
apparatus which aid in directing the user to the internal
location of information with which the book is concerned;
these include, in descending order of size, the index,
the table of contents, and the title. The library
itself is of course a collection of books but it too
contains certain apparatus for directing the user to
those amongst the many books held that contain information
concerning some particular matter; these include, in
increasing order of size, the classification system,
the reference section, and the card catalog. There
are also other types of traditional access means that
aid in locating books which contain certain information,
including special bibliographies and, too often overlooked,
the reference librarians. If indeed size is the pre-
dominant factor determining the need for access, then
a study of the size of the various natural bibliographic
units named above may shed light on the structure, if
any, of the traditional access systems and thereby
also provide guidelines for those who study the possible
ways for increasing and automating the means of access.

We will proceed up the scale of size of the naturally
occurring access means associated with books and collec-
tions of books, with the intent of determining the
statistical distribution of size of each such
system; this information will lead in a natural way to
the level structured model of access systems briefly
described in Chapter I.

Initially limiting our attention to the book itself,
there are four systems of interest:

      1.   Title

      2.   Table of Contents

      3.   Index

      4.   Book Text

In each case we wish to know the mean (average) <u>size</u>
of the item in question, measured, let us say, by the
number of characters (including the interword space)
contained in the item. Moreover, it will turn out to
be important to know the distribution of size for each
case so that it will be possible to say to what the
extent the mean is characteristic of the distribution
and also because the distributions will turn out to
have an intrinsic connection with the access problem
via the intervention of the mathematical discipline
known as <u>information theory</u>; this latter aspect of our
study will be described in Chapter III.

It is not easy to obtain reliable statistics about the
size of bibliographic units; it is especially difficult
if general samples that are not restricted to one or
a few fields of interest are desired. We have based
our book studies on the <u>Fondren Sample</u>, a random sample
of 1926 cards drawn from the shelf list catalog of the
Fondren Library at Rice University in 1968; it has been
described in some detail in Ref. (1). Associated with
each shelf list card is one or more monographs; these
monographs constitute the sample on which our study is
based. It is appropriate to refer to it as a <u>random</u>
<u>sample of books from a medium sized university library</u>.

Because we are interested in studying the interaction
of the various traditional access systems used in books
we have extracted from the Fondren Sample all those
books that contain an <u>index</u> (here and throughout all
that follows, <u>index</u> will of course mean <u>back of the</u>
<u>book index</u>), thus yielding what we have called the
<u>Fondren Index Sample</u>, which may reasonably be called a
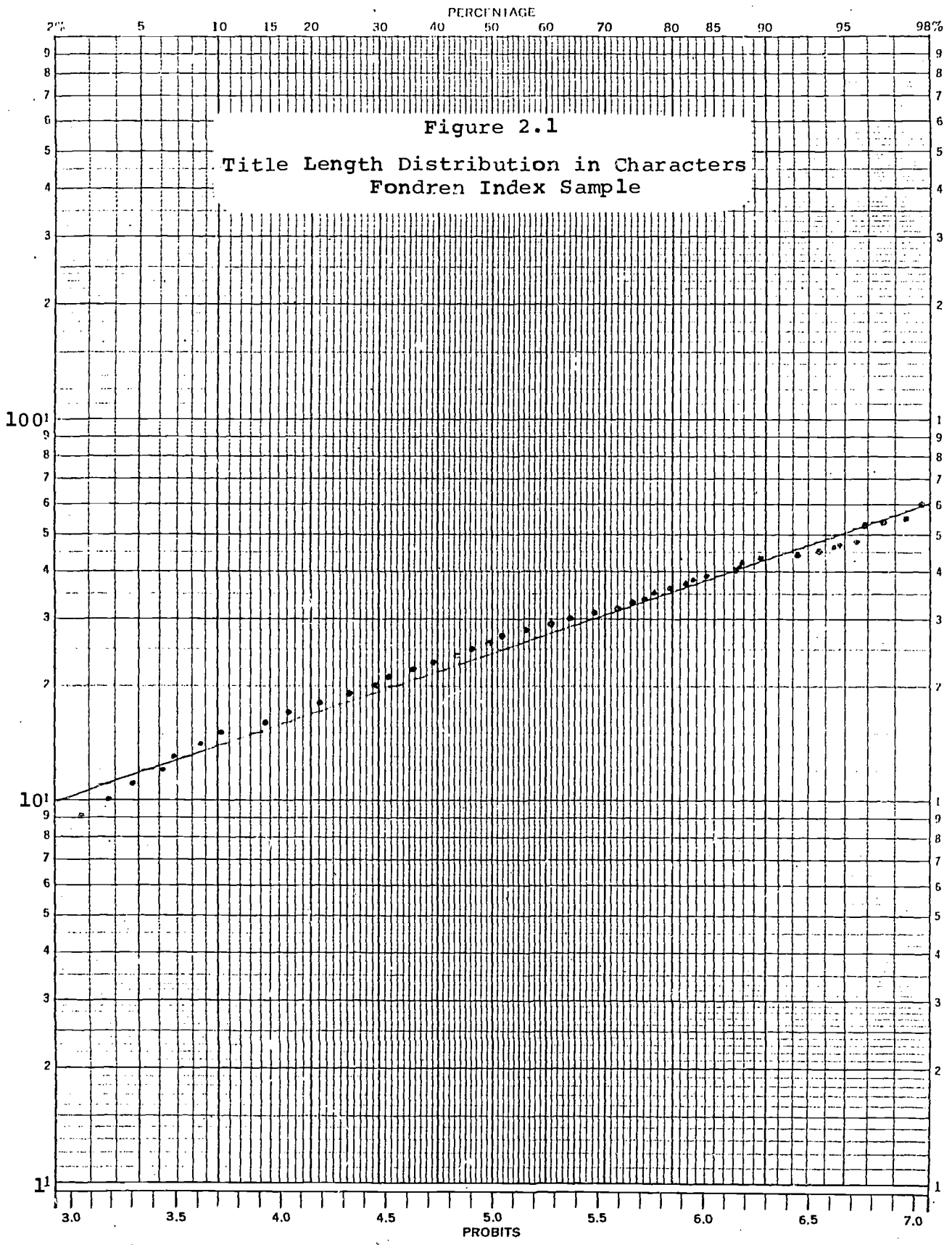<u>random sample of indexes</u>. There are of course certain

unavoidable biasses present in this index sample: the Fondren Library does not have an adequate collection in medicine or law, for instance; it has an exceptionally fine collection in other areas. But, to the best of our knowledge, these samples are the closest in existence to truly random samples of books and of books with indexes belonging to the complete population of all books ever published.

With these preliminaries in mind we can now turn to study the structure of book titles. Figure 2.1 displays the distribution of the number of characters per book title for books from the Fondren Index Sample drawn on lognormal probability graph paper. The mean number of characters per title is 28.15.

Next consider the size of a table of contents measured by the number of characters it contains.

Although the "structure" of a book title is relatively standardized, the same cannot be said of the table of contents. Some books include phrases such as "Chapter 1", others simply record "1" to designate the first chapter, and others do not bother to indicate the chapter ordinal at all. There are tables of contents which include, in addition to a chapter title, relatively extensive descriptions of the text content of a narrative nature; others include section titles. Despite the rather excessive degree of variation that does occur, there are certain components of a table of contents which appear to be nearly invariable in their presence, including the chapter titles and page number designating the beginning of each chapter. We have chosen to define the table of contents as that portion of the material contained in what is normally termed the table of contents that corresponds to the chapter title, excluding from consideration all headings, chapter ordinals, appendices, tables of figures, etc., and page number referents to the location of chapter initial pages. With this convention, a random subsample of 161 tables of contents was selected from the Fondren Index Sample and the number of characters (including interword space characters) was counted for each selected table of contents. It turns out that the mean size of a table of contents defined in this way is 505 characters. Figure 2.2 displays the distribution of table of contents size for this subsample.

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Figure 2.1

Title Length Distribution in Characters
Fondren Index Sample

FIGURE 2·2

DISTRIBUTION OF
SIZE OF TABLE OF CONTENTS
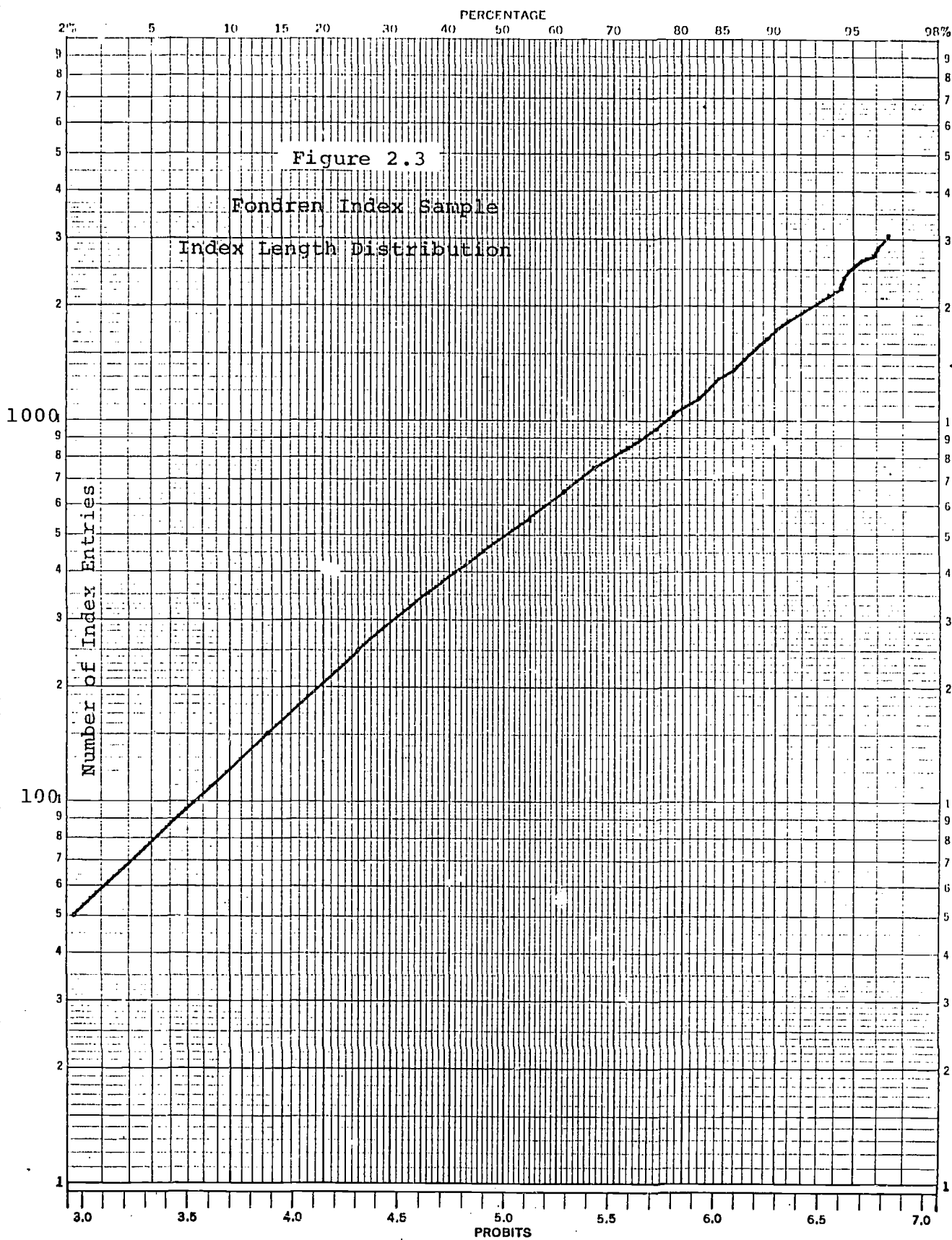FOR BOOKS IN THE
▽FONDREN INDEX SAMPLE▽

The reader can hardly help but notice that the data
exhibited in each of Figures 2.1 and 2.2 fall nearly
along a line, and moreover that the two lines have
similar slope. The graph paper is so designed that
straight lines indicate that the data are drawn from a
lognormal distribution, whose properties will be dis-
cussed later on in this chapter and extensively in
Chapter III; it suffices here to stress that thus far
the data indicates that the two lowest levels of distri-
bution of size of book access systems belong to some
well known family of statistical distributions and
indeed to the same family. We will want to look for
this possibility when examining data referring to other
access systems.

The index is the next largest access tool traditionally
found in books, and from many points of view it is the
most important and responsive to the detailed demands
of the user. It therefore deserves extensive examination.

The Fondren Index Sample consists of 706 indexes.
Chapter IV investigates the relationship of indexed
books to the unindexed books in the Fondren Sample and
studies such properties of the indexed books as their
distribution among the Library of Congress classifica-
tion categories. Here we are only interested in
considerations of size. The mean number of index
entries per index is 836.

Figure 2.3 contains the distribution of the number of
index entries per book, again on lognormal probability
graph paper. It is evident that the data can be accur-
ately approximated by a line and furthermore that the
line has a slope which once again is similar to the slope
of the lines occurring in the previous two figures.
One word of caution: here only the number of index
entries is exhibited. Ideally one would wish to
measure the size of an index by the number of characters
it contains, but it would not be feasible to count the
characters in more than half a million index entries.
Furthermore, once again the question of which characters
to count can not be resolved in a completely unambiguous
way. For instance, it is easy to agree whether page
reference numbers should be counted, and what to do about
consecutive spaces used as separators, but format problems
related to multiple entries grouped under a common
initial phrase, and inverted order entries demand opera-
tional decisions that are not often guided by a clear
cut purpose. These problems exist when entries alone
are counted, but they are magnified when characters are
counted. We have agreed, when counting entries, to count

24

Figure 2.3

Fondren Index Sample

Index Length Distribution

each group of page reference numbers: this defines
the index entries, at least as far as their cardinal
number is concerned, and provides a relatively clear
cut procedure requiring a minimum amount of subjective
decision by the persons performing the counting. In
order to obtain an approximation to the number of char-
acters contained in an index, a rather indirect procedure
was used. We have in a convenient form all of the index
entries contained in 80 books in the field of statistics,
all printed in a fixed typefont whose characters are of
constant width, and printed a fixed number of lines to
the page. These characteristics make it possible to
count the number of characters in an entry by measuring
the length of the entry. This was done for a uniform
subsample (comprising about 1.75% of the total Statis-
tical Index Sample of 31,232 index entries). Table 2.1
lists the number of entries consisting of from 1 to
76 characters, and, opposite 77 characters, the number
of entries that had at least 77 characters. The mean
number of characters per entry, exclusive of page
reference numbers but inclusive of interword spaces, is
25.47. Figure 2.4 displays the distribution of
size of the entries in the Statistical Index Sample.
If we assume that the distribution of size of index
entries is independent of the distribution of the number
of entries per index, then the average number of charac-
ters per index will be the product of the average number
of entries by the average number of characters per
entry. Using the number for the Statistical Index
Sample for the latter, we find that the average number
of characters per index (exclusive of page references)
is 836 x 25.47 = 21,293. If it be assumed that there
are typically three digits and an interword space
required to provide the page reference location informa-
tion, then augmenting the average number of characters
per entry by 4 leads to 24,560 characters per index
(inclusive of page reference approximation).

The distribution of index entry length for the Statistical
Index Sample is, again, lognormal to a high degree of
approximation.

Table 2.1

STATISTICAL INDEX SAMPLE

Distribution of Entry Length in Characters
(excluding page references)

No. of Char.	No. of Entries		No. of Char.	No. of Entries
1	0		41	3
2	0		42	3
3	4		43	6
4	2		44	3
5	4		45	5
6	7		46	5
7	8		47	3
8	9		48	4
9	20		49	4
10	22		50	2
11	19		51	4
12	18		52	4
13	14		53	3
14	23		54	6
15	11		55	4
16	28		56	3
17	16		57	2
18	8		--	
19	14		62	1
20	12		63	1
21	17		64	1
22	17		65	1
23	19		--	
24	19		67	1
25	14		--	
26	8		69	1
27	11		--	
28	9		72	1
29	11		73	1
30	7		--	
31	10		75	1
32	9		76	2
33	10		≥ 77	9
34	6			
35	5			
36	4			
37	5			
38	10			
39	10			
40	9			

27

35

Figure 2.4

Distribution of Entry Length

(in Characters)

Statistical Index Sample

The last of the four natural access tools for monographs
is the monograph text itself. It will be even more
difficult to estimate the size of a book measured by
the number of characters it contains because of the
variability of type form and page layout supplemented
by the presence of tabular and figured material. Al-
though numerous different and justifiable procedures
of making such a size estimate are conceivable, we have
once again attempted to choose a method that would be
simple and insensitive to subjective judgements of the
personnel performing the task in order to improve
accuracy but more importantly to make it possible for
other workers to reproduce (at least nearly) our
results. Regarding book text, there are several levels
of analysis that require an increasing amount of
extraneous and unstandardized information. The simplest
measure, and one that it easily reproduced, is simply
to transcribe the arabic number shown on the catalog
card designating the number of non-front matter
pages. It is difficult to say precisely which pages
are represented by that number in each case, but it is
unnecessary to do so; we simple agree that this number
defines the length of the book in pages. The distri-
bution of book length measured in pages was determined
in Ref. (1) for the complete Fondren Sample. The mean
number of pages per book is 276.6; the distribution
of pages is however not lognormal as is readily seen
in Figure 2.5. If the corresponding distribution is
plotted just for those books that do have indexes (i.e.,
for the Fondren Index Sample), the graph in Figure 2.6
results, which shows that the distribution of size of
these books is lognormal. This suggests that there
may be some intrinsic structural difference between
books which contain an index and those that do not.
If attention is restricted to the Fondren Index Sample,
it turns out that the mean number of pages per book
is significantly greater, namely 341.5. The next step
in determining the number of characters per book is to
find the number of lines per page and their length;
this has been studied by Dolby and Jones (Ref.(2)),
who found 38 lines of 24 picas as the mean. The final
step in obtaining an estimate of book size in characters
is to approximate the number of characters per 24
pica line of print; we have analyzed a sample of printed
matter and find 63 characters per 24 pica line as the
mean. These estimates together imply that an average page
of printed text contains 2394 characters, including
interword and end of line spaces. Hereafter it will
be assumed that there are 2400 characters per page. We
have no idea what the effect of tabular and figured
material as well as other formatting conventions is on

FIGURE 2.5

DISTRIBUTION OF BOOK LENGTH IN PAGES

FOR ALL BOOKS

FROM THE MFONDREN SAMPLE

PERCENTAGE

PROBITS

NUMBER OF PAGES

K+E PROBABILITY X 3 LOG CYCLES KEUFFEL & ESSER CO. 46 8080 MADE IN U.S.A.

30
38

FIGURE 2.6

DISTRIBUTION OF BOOK LENGTH IN PAGES
FOR BOOKS WITH INDEXES
FROM THE "FONDREN SAMPLE"

the estimate of book length in characters; nevertheless, excluding these matters from consideration, we find that the average book in the Fondren Index Sample is 341.5 x 2400 = 819,600 characters in size.

Turning now to collections of books, let us first consider the university library. Here it is essential that the notion "university" be specified in some way so as to enable one to distinguish university libraries from libraries of colleges in a manner consistent with that used for other purposes by governmental agencies and the educational institutions themselves. We implicitly use the definition used by the Office of Education of the Department of Health, Education and Welfare because we use their statistical data book Reference (3) as our source of information about the holdings of college and university libraries.

Unfortunately the data presented in Reference (3) is incomplete; notable ommissions are the University of Chicago and Yale University. Although these ommissions undoubtedly will have some influence on the statistical parameters of the distributions of interest to us, these will most likely be quite minor and in no event can they be expected to change the form of the distribution nor substantially affect its mean or variance.

There is one other defect of the data presented in Reference (3) which is more critical for our concerns. Most state university systems have had their statistics amalgamated; thus it is impossible to determine (from this source) the size of the library of the University of California at Berkeley--only the total number of volumes held in the entire California university system is presented. This unfortunate state of affairs holds for most of the other state systems also and tends both to depress the number of distinct university libraries and inflate the size of those that remain. Two factors permit us to extract useful information from this tabulation despite its amalgamated nature: first, it is easy to obtain lists of all units belonging to a state system (and also for the few private systems that operate more than one campus) and thereby estimate the total number of libraries whose structure must be studied. Second, within state systems there is usually one 'giant' library and a number of much smaller ones; this has the consequence that the departure of the distribution from lognormality, as is shown in Figure 2.7 which we will shortly consider, is diminished when the separate system units are accounted for, and, in view of the smallness of the possible effect, it is not necessary for us to study this difference in detail. Furthermore,

we can easily obtain the mean size from the revised
estimate of the number of libraries. By adjusting the
number of libraries represented in Reference (3)
through deletion of the special dental and medical
school branches and addition of all general campuses,
a total of 201 university libraries is attained.
The total number of volumes held in these institutions
is 152,230,163 (nearly one for every inhabitant of the
United States, and nearly as many as are held by all
public libraries), so the mean number of volumes per
university library is 757,364. The range in size may
appear remarkable to the reader, ranging as it does from
some 100,000 volumes to more than 8 million. Figure 2.7
exhibits the size distribution, which, as we have by
now come to expect, is lognormal.

Knowing that the average book contains 819,600 characters
and assuming that the distribution of book size is
independent of the distribution of university library
size, we readily find that there are some 620,735,534,400
= 6.2×10^{11}, or approximately 620 billion characters
stored in the average university library.

At this point we have established the mean size and
distribution of size for book based bibliographic enti-
ties ranging in average size from about 30 characters
up to 620 billion characters, entities which differ
in size by a factor of 20 billion. Our immediate task
is to demonstrate that there is a simple and reasonable
model which encompasses the entire range of biblio-
graphic entities in a systematic way, relating those of
one size to those of another in a uniform and un-
varying manner.

In order to proceed, recall that the book title, table
of contents, index, and text are four bibliographic
units of increasing average size; let us say that
they belong to levels 1,2,3,4 respectively. Let Y_n
stand for the base 10 logarithm of the average
size of the units belonging to level n; Figure 2.8
displays the points whose coordinates are (n, Y_n) for
n = 1,2,3,4, and also the point $(8, Y_8)$ where Y_8 is the
base 10 logarithm of the mean size of a university library,
and the point $(7, Y_7)$ where Y_7 is the base 10 logarithm
of the mean size of a two-year college library, obtained
by analyzing the first 206 two-year college libraries
listed in Reference (3); this procedure is biassed,
leading to a slightly high estimate of the mean size of
two-year college libraries because the State of California
dominates the initial part of the list both in number
of two year colleges and in the size of their libraries,

33

Figure 2.7

DISTRIBUTION OF
SIZE OF UNIVERSITY LIBRARIES

(Source: "Library Statistics of
Colleges and Universities", Fall 1969)

Figure 2.8

THE LEVEL STRUCTURE
OF ACCESS SYSTEMS

BASE 10 LOGARITHM OF SIZE

• University Library

• Two Year College Library

• Text of Indexed Book

• Index

• Table of Contents

• Title

LEVEL

but analysis of the complete list in Reference (3),
which is presently underway, will undoubtedly lower the
mean size insignificantly from the value 29,912
volumes used to determine the corresponding point in
Figure 2.8.

Figure 2.9 confirms that the size distribution of two
year college libraries is lognormal and that the slope
of the line representing the data on that graph is once
again comparable with the slope of lognormal distri-
butions presented in previous figures in this Chapter.

Inspection of Figure 2.8 may lead the reader to wonder
whether levels 5 and 6 correspond to naturally occurring
collections of books; we think that level 5 corresponds
to general encyclopedias and level 6 to personal libraries,
but we have not ventured to include calculations based
on these hypotheses because of the difficulty of
amassing reliable and comprehensive statistical
information in their support.

The points in Figure 2.8 evidently lie very nearly on
a straight line. This means that the mean size, s(n),
of the bibliographic units comprising the n-th level
is related to n by an equation of the form

$$s(n) = a10^{bn} \qquad\qquad (2.1)$$

where a and b are constants. It is natural to suppose
that a = 1 so that level 0 corresponds to the single
character; we will examine the data given in Figure 2.8
and Table 2.2 which corresponds to it to see if it is
consistent with this desirable and simplifying
hypothesis. By a standard application of the

Figure 2.9

SIZE OF TWO YEAR COLLEGE LIBRARIES

(SOURCE: "LIBRARY STATISTICS OF
COLLEGES AND UNIVERSITIES", FALL 1969)

Table 2.2

SIZE IN CHARACTERS

OF VARIOUS BIBLIOGRAPHIC UNITS

Unit	Level	Size	Log_{10} of Size
Title	1	28.15	1.44948
Table of Contents	2	505.	2.70329
Index	3	21293.	4.32710
Text of Book	4	819600.	5.91360
Two Year College Library	7	24528169200.	10.38966
University Library	8	620735534400.	11.79291

statistical F-test, as described for instance in Ref. (4), it is easily shown that the data does not contradict the hypothesis that a = 1 in eq. (2.1) at the 5% confidence level; this means that the least squares best fitting line for the points in Figure 2.8 does not differ significantly from that line which is constrained to pass through the origin of the coordinate system and also minimizes the sum of the squares of the deviations from the data points. This latter line corresponds to a relation of the form

$$s(n) = 10^{bn} \qquad (2.2)$$

relating the mean size of bibliographic units to their level. Carrying out the least squares minimization for a function of this form on the logarithms of the data leads to the line drawn in Figure 2.8 which corresponds to the equation

$$s(n) = 10^{1.47247n} = (29.68)^n. \qquad (2.3)$$

The constant 29.68 is an estimate of the fundamental
constant determining the level structure of the bibliographic units considered above. More extensive data
will no doubt result in the modification of this value,
but it can be said with certainty that the fundamental
constant is approximately 30, and perhaps may be
identifiable with $(2e)^2 = 29.54...$, where $e = 2.718...$
is the mathematical constant denoting the base of the
natural logarithm system.

This is our first main result:

> The average size of the bibliographic units
> <u>title</u>, <u>table of contents</u>, <u>index</u>, <u>monograph</u>, <u>two</u>
> <u>year college library</u>, and <u>university library</u> are
> powers of a fixed constant K whose value is
> nearly $(2e)^2$.

If it could be shown that the mean size of an encyclopedia is approximately K^5 and that of a personal (or
perhaps a library reference sublibrary) is about K^6,
then it could be asserted that the natural bibliographic
units are <u>equispaced</u> when measured by the logarithm
of their size; the current state of knowledge only permits
us to assert that this is so for levels 1 through 4
and also for the separation of levels 7 and 8.

The previous argument suggests that the notion of <u>level</u>
be introduced more generally. Therefore define the
<u>level</u> of a given information base to be the integer
closest to the logarithm of its size (the latter measured
as usual in characters) to the base K; moreover, if
a system of level K provides access to an information
store of level n, then define the <u>order of access</u>
provided by the access system as (n-k). Thus an index
provides access of order 1 (=4-3) to the monograph
it accompanies, and similarly the table of contents
and title provide access of order 2 and 3 respectively
to the book with which they are associated. We will
later find that a library card catalog provides access
of order 2 to the library archive but unfortunately it
occupies a physical volume which could provide order 1
access to the collection.

Thus far we have principally concerned ourselves with
the mean value of the various size distributions that
have been examined, and have thereby shown that there
is a simple and uniform relationship which connects
the smallest of the natural units to the largest. We
must now take up the question of the extent to which the
mean characterizes the distributions that occur.
The figures displaying the various distributions at
the same time provide powerful evidence that all of the

distributions are lognormal. The elementary form of
the lognormal function, which is what occurs here,
depends on two parameters--the lognormal mean and the
lognormal standard deviation; if these parameters are
known, then the usual mean value of the distribution
can be determined and conversely, if the lognormal
standard deviation and the usual mean are known, the
lognormal mean and hence the lognormal function itself
are completely determined (cp.Chapter III). From this
it follows that if the lognormal standard deviation of
the various distributions of interest are all essentially,
equal, then the associated lognormal functions are
in reality determined by the mean value, that is, by
the level, of the distribution. We shall show that this
is indeed the case. Table 2.3 lists the lognormal
standard deviation of the six distributions that have
been described thus far.

Table 2.3

LOGNORMAL STANDARD DEVIATION

Unit	Level	Lognormal S.D.
Title	1	0.19
Table of Contents	2	0.30
Index	3	0.44
Monograph	4	0.23
Two Year College Library	7	0.29
University Library	8	0.36

There is evidently not much variation of the lognormal
standard deviation as the level changes from a distri-
bution whose typical size is about 30 characters to one
whose typical size is about 600 billion characters and
in particular what variation there is does not seem to
have a trend. Based on the data contained in Table 2.3
we assert that the lognormal standard deviation is
essentially constant throughout the entire range of
bibliographic interest, and consequently the distributions
of size of the various bibliographic units are determined
by the level of the unit.

The lognormal standard deviation corresponds to the slope of the line defining the lognormal function for figures drawn on lognormal probability graph paper such as Figures 2.1-2.7 and 2.9 are. The underlined statement in the previous paragraph is the analytical version of the geometrical assertion that the lines representing all of the distributions are nearly parallel. We show to what extent this is so in Figure 2.10 which displays the distributions for all six levels; the variation of slope is indeed not great. The mean value of the standard deviations listed in Table 2.3 is 0.30, which may be conveniently adopted as an estimate of the level-independent lognormal standard deviation.

The assertion that the distribution of a variable x is lognormal is equivalent to stating that the distribution of log x is the normal (Gaussian) distribution. Here 'log' denotes the logarithm with respect to any conveniently chosen base. The graph of a normal distribution is the well known 'bell-shaped curve'. The level-structured lognormal distribution model of access systems described above can be equivalently viewed as a level-structured model for the logarithm of the size of bibliographic units such that the mean of the logarithms of the various levels are equally spaced and the associated distributions are normal, as shown in Figure 2.11 for levels 1-3. From that figure one also sees that the several bell curves have little overlap; this corresponds to the relative horizontality of the lines in the previous Figure 2.10 which is another way of stating that the lognormal standard deviation is a small number. The converse possibility, which fortunately does not occur, is that the lognormal standard deviation be relatively large with the consequence that the normal distributions like those illustrated in Figure 2.11 would possess a large degree of overlap with the overall appearance of gentle waves uniformly spread over a sea rather the sharply defined and separated peaks and valleys that Figure 2.11 so clearly exhibits. What this means is that the notion of level for bibliographic units makes sense; almost all units of some given type are of a size that is closer to the level of that type than to any other level. For instance, from Figure 2.10 we can read that fewer than 0.05% (sic!) of the Tables of Contents are so large as to lie (in logarithmic measure) closer to level 3 (Indexes) than to level 2 (Tables of Contents); similarly, fewer than 0.2% of the Two Year College Libraries are so large that they lie closer (in logarithmic measure) to the average size of a university library than to the average size of a two-year college library.

FIGURE 2.10

DISTRIBUTION OF
SIZE OF BIBLIOGRAPHIC UNITS

UNIVERSITY LIBRARY

TWO YEAR COLLEGE LIBRARY

MONOGRAPH

MONOGRAPH INDEX

TABLE OF CONTENTS

MONOGRAPH TITLE

LOG₁₀ OF SIZE

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50

FIGURE 2.11

SOME ACCESS DISTRIBUTIONS IN
LOGARITHMIC VARIABLES

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These observations suggest that the notion of boundary separating two adjacent levels should be introduced as that size corresponding to half integer values of the level. More precisely, with level n and size s(n) related as in eq. (2.2), we say that the size s(n+1/2) is the boundary size between s(n) and s(n+1), and that (n+1/2) is the boundary between level n and level (n+1).

With this notion in hand it becomes possible to analyze a bibliographic item in order to determine if its size coincides reasonably with its 'proper' size, i.e., with the level of that type of bibliographic unit; from its size s compute $\log_K s$ and compare this number with the appropriate bibliographic unit level n to see whether $\log_K s$ lies within $\pm$ 1/2 of n; if it does not, then we may assert that the item of size s is either too large or too small. There will of course be specific exceptional instance for which the size of the unit is indeed 'proper' although not consistent with the statistically typical behavior for items of its bibliographic type, but the designer or evaluator of information access systems and/or information bearing data bases should, we think, warily approach the question of the size of a system from this point of view.

The access model presented in this chapter is not restricted to the book and its subsystems and super-systems. There is considerable evidence that it reflects universal properties of information sto d in written English form, and, in a slightly genera zed version, may be still more broadly applicable ʃ che analysis and modeling of other types of inform.cion systems such as those associated with the modalities of sensory perception. These wide ranging and difficult issues cannot be examined here in a serious way; more-over, we do not yet have sufficient data upon which a definitive report can be based. Some of the intriguing vignettes that are most directly related to information presented in forms analogous to, if superficially distinct from, the book information system hierarchy explored above may nevertheless prove helpful for the reader.

First consider the size relationships of component units of the serial publication archive. We have studied the mathematics journal subarchive with the following results. For 7445 papers reviewed in volume 36 of Mathematical Reviews (published in 1968), the mean length of an abstracted paper is 13.8 'pages'; here 'page' refers to the myriad distinct page sizes and formats used by the 800-odd distinct journals reviewed by Mathematical Reviews. Bearing this in mind, and

noting that we have not attempted to directly determine
the mean number of characters per page of mathematics
text nor the effect of the numerous special symbols
which extend the normal type font, use of our previous
estimate of 2400 characters per page of text yields
the estimate of 33,120 characters per mathematics paper;
hence such a paper is of level 3.   The mean length
of an abstract in Mathematical Reviews is easily
estimated to be about 1081 characters.   Therefore the
size of the average mathematics paper is 30.6 times the
size of the average abstract.   Division of the esti-
mated size of an abstract by $K = 29.54$ gives 36.59
characters, which is about the size of the average
mathematics journal paper title and is of course quite
close to the level 1 mean of 29.54 characters.   We
conclude that journal papers in mathematics are
structured in a manner which is consistent with the
general model proposed for books.

Next consider a more complex example which refers
directly to the access problem.   It is usual to find
so-called "subject headings" at the foot of library
catalog cards which are intended to provide cross
reference access to subject areas other than those
associated with the class number of the item corresponding
to the catalog card.   There are nearly 93,000 subject
headings in the Library of Congress Subject Headings,
seventh edition (1966).   A uniform 1/66 sample drawn
from an alphabetized list of these headings shows that
the mean number of characters per subject heading is
22.3, which is not remarkably close to $K = 29.54$.
However, the distribution of subject headings per catalog
card as determined from an analysis of the Fondren
Sample has a mean of 1.2 headings per card; if the
distribution of subject headings per card is independent
of the distribution of characters per subject heading,
then the mean number of subject heading characters per
catalog card, including the associated ordinals and
interword space characters, will be the product of the
means of the component distributions, which is 29.16.
Hence the collection of subject headings per card
provides about the same level of discrimination above
the one-letter Library of Congress class in the mean
that is provided by the title.   Considering the dis-
tributions of characters per subject heading and subject
headings per card leads to the lognormal functions shown
in Figure 2.12; we conclude that the subject heading
access mechanism is consistent with the level structured
model and it belongs to level 1.

Figure 2.12

Distribution of Number of
Characters per LC Subject Heading

Distribution of Number of
Subject Headings in Fondren Sample
Excluding Serials

The phenomenon that the mean value of the size of adjacent access levels are in the ratio of about 30 to 1 is not confined to access systems associated with written natural language archives. Consider ALTEXT, a contemporary text-processing higher level (macro expander) computer language [5]. Such a language consists of computer instructions which have two parts: a generic instruction such as the GOTO of FORTRAN which specifies the general function of the instruction, and certain other more particular components which contain the details of data location and transfers of control. The implementation of a higher level computer language instruction consists of a sequence of one or more "machine language" or "assembly language" instructions; the advantage of the higher level language is that it frees the programmer from the burden of keeping track of numerous housekeeping details concerning the location and manipulation of the data at the cost of lower (local) efficiencies of execution. This is another way of stating that the higher level language instructions act as an access system for the sequences of assembly language instructions that are their implementation.

With this preamble in mind, one can examine the number of assembly language instructions required to implement each of the distinct generic higher level language instructions. For the generic instructions of ALTEXT, the mean number of assembly language instructions per ALTEXT "macro" is 30.?2 (including implementation of the "ALTEXT macro" which provides the interface with the operating system of the implementing computer) for implementation on the IBM 360/30 computer. Figure 2.13 confirms in a rather startling way that the distribution of implementation size is lognormal; hence we conjecture that the level structured access model will probably find significant application in the design of computer languages.

That the structure of many types of linguistic units is lognormal has long been known and abundantly verified. The lognormality of word length statistics was discovered at least as early as 1887 by Mendenhall [6] and was subsequently studied, along with sentence length distributions, inter alia, by Yule [7], Williams [8], and Herdan [9]. Yule computed the sentence length distributions for a number of samples of written English and although he did not notice their lognormality himself, Williams did test this hypothesis on Yule's data and on more he gathered himself. More extensive data has been collected by Kucera and Francis [10] but care must be exercized to insure that it is partitioned into homogeneous subject and/or author classes before attempting to study the lognormality of the statistics; the problem

of describing the structure of inhomogeneous data, which amounts to studying how distinct lognormal distributions combine, is relatively complex. Moreover, much of the Kucera and Francis data refers to printed materials that are unlikely to form an active part of an archival library collection; it is heavily weighted with fiction and press coverage.

Herdan |9| analyzed 80,000 words of telephone conversations collected by French, Carter and Koenig of the Bell Telephone Laboratories and concluded that (phonetic) word length is lognormally distributed. An indication that the parameters of these linguistic distributions are relatively insensitive to variations in language vocabulary and to whether the written or spoken form is used is provided by Figure 2.14 which shows nearly parallel lines representing the Herdan telephone conversations and Mendenhall's analysis of 1000 words from Shakespeare's works (as represented by Williams).

These examples and others too numerous to report here prompt us to speculate that the occurrence of the lognormal distribution is fundamental to all human information processing activities. In this regard we distinguish two types of activities: those that process direct sensory impressions that are received through the sensory organs, and those that process coded information such as is represented by linguistic codes. In the latter instance the directly perceived data arrives via the sensory organs but the essential content is unrelated to the particular code used for its transmission. Although there may be important differences between the internal mechanisms that process these two types of information, there are at least two characteristics that the two types of input information share: the quantity of information that passes through the processing system is very large and the system must be capable of responding to inputs whose size vary greatly. The first condition requires that the information processing system be able to compress (with information loss) the vast amount of data passing through it so as to be enabled to retain for future use a much smaller but characteristic subset of it; in other words, the processing system must function as an access system to the information passing through it. The second condition suggests that some functional transformation must be applied to the input sensory information in order to reduce its extended range to a smaller one more conveniently handled by the neural network; for example, there has long been evidence (which is reflected by the 'decibel' scale of measurement) that the subjective response to the stimulus provided the ear by acoustic energy varies as the logarithm of the input energy.

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Figure 2.13

ALTEXT macros ranked by
number of assembly language instructions
in IBM 360 implementation
for postulated 33 1/3 macro language

ALTEXT macro

Figure 2.14

Word Length Distributions
(in characters)

Generally, there are three reasons for making a scale
transformation in analyzing data (e.g., see Tukey [11]):

1. To linearize the relation between two variables.

2. To normalize the underlying probability
    distribution.

3. To stabilize the variance.

Although in most applications any one of these results
would provide sufficient reason for introducing a
particular transformation, it is not uncommon to encounter
situations where the transformation is originally
introduced for one reason and subsequent analysis shows
one or both of the remaining desiderata have also been
achieved.

In this context it is illuminating to study the work
of the nineteenth century experimental psychologist
G. Fechner [12]. He made the important observation
that the ability of the human to respond a stimulus
is proportional to the mean level of the stimulus.
That is, if an individual can just sense a difference
of, say, one unit when the mean level of stimulation is
10 units, then he will also just be able to detect a
difference of 2 units when the mean level is 20 units.
This multiplicative property of the just noticeable
difference led him to introduce the logarithm function
in order to stabilize the variance, i.e., make it
constant throughout the range of perception. He then
conjectured that the function relating subjective
response to the transformed variable--the logarithm of
the stimulus--is a linear function, thus arriving at
the celebrated (and once again hotly debated) 'Law'
of Weber and Fechner. The reader will observe that the
logarithm of the size of bibliographic units stabilizes
the variance of the distributions of these units through-
out the entire range of 'bibliographic perception'.
This certainly makes it tempting to inquire whether the
Weber-Fechner 'Law" might not be merely an approximation
to some more accurate description of the underlying
functional transformation governing sensory perception.
This question has received considerable attention in
recent years and notable contributions have been made,
principally by Stevens (e.g., [13]), who has generalized
the logarithmic Weber-Fechner transformation so that
response is some power of stimulus; that this change
actually constitutes a generalization becomes clear when
it is noted that the integral of $1/x$ is $\log x$ whereas
the integral of any other power of x is again a power
of x; in this sense the logarithm is the limit of power

functions (see Dolby [14]). The relationship between
linguistic and hence bibliographic units and these
psychophysical questions has been remarked by several
workers, most notably perhaps by Fairthorne [15];
Zipf's 'Law' [16] in its integrated form is just the
Weber-Fechner logarithmic relation, and Mandelbrot's
[17] generalization of Zipf's function corresponds--
indeed, it is identical to--Steven's power function.
These questions will be taken up from a more mathematical
standpoint in the next chapter with the intent of
showing how they can be derived, following an argument
essentially due to Mandelbrot, from elementary
considerations from information theory, and, of
more importance for our purposes, that a slight exten-
sion of this argument generalizes the Weber-Fechner-
Zipf-Stevens-Mandelbrot functions to the lognormal
distribution. For as the extensive bibliographic
data assembled in the earlier parts of this chapter
show, it is the lognormal function that in fact describes
reality.

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CHAPTER III

MATHEMATICS OF

INFORMATION DISTRIBUTIONS

MATHEMATICS OF
INFORMATION DISTRIBUTIONS

This chapter is devoted to the mathematical study of
some of the distributions that arise naturally in the
study of information systems.   It will necessarily
be more demanding of the reader's mathematical knowledge
than the remainder of the book and has therefore been
written so as to permit the reader to pass immediately
to Chapter IV without loss of continuity. We believe,
however, that the significance and implications of the
level structured model of access systems presented
in Chapter II cannot be fully understood unless the
relationship of that model to other competing models,
extant and potential, is made clear.   Moreover, the
most powerful theoretical arguments for the appear-
ance of the lognormal distribution in the model struc-
ture come from information theory and its mathematical
apparatus, so there is really no way to avoid these
technical considerations.

We will be principally concerned with two distributions--
the Zipf and the lognormal.   The former is also frequently
associated with the names Estoup (1), Bradford (2),
and more recently Mandelbrot (3,4).   Zipf rediscovered
and popularized the observation that the ranked frequency
distribution of words in natural text corpora is essentially
of the form

$$y = cx^{-s} \qquad\qquad (3.1)$$

where x denotes the rank and y(x) the frequency of
occurrence of the word of rank x; here c and s are
constants selected to fit the data as nearly as possible
and which therefore are characteristic of the text
corpus and to some extent the language from which it
is drawn; Zipf only considered the case s = 1.
Figure 3.1, taken from Zipf (5), exhibits such distributions.

Distributions of the type (3.1) occur in other fields,
associated, for special values of s, with Pareto (6)
in economics, Lotka (7) in what might be termed
'sociological mathematics', and more recently De Solla
Price (8), and no doubt in numerous other contexts
as well.

That the Zipf "law" is taken seriously, not just considered
as an accidental quirk of the data to be remarked upon
and ignored, is attested by the variety of publicatons
that dispute, modify, and reduce it to a triviality.
Mandelbrot showed that a slight generalization of the

Figure 3.1

Zipf law, in better agreement with the data, is a con-
sequence of elementary arguments and reasonable hypotheses
about the effort required for the efficient transmission
of information; in this sense he is a bulwark for both
the proselytizers and trivializers of Zipf since
his arguments convincingly show that the nature of the
distribution has nothing to do with special properties
of language that distinguish it from a variety of other
processes that extremize some function representing
the degree of organization of the process in a statisti-
cal sense. Mandelbrot was quick to point out the con-
nection, which is more than merely formal, between
his result and the mathematical methods used to derive
it, and the derivation of the partition function in
statistical mechanics; cp. Schrödinger (9).

Despite the considerable research efforts that have
gone into understanding and improving the relation
of Zipf, there are significant discrepancies between
the Zipf-Mandelbrot predictions and the observed data
for large samples of words drawn from natural language
text corpora and for other data collections as well.
There are theoretical difficulties too: Yule (10)
observed that the sum of the frequencies predicted
by the Zipf-Mandelbrot distribution (with s = 1) is not
finite, which implies that there must be a significant
deviation from this distribution for large values of
the variable. This kind of difficulty is not as easily
brushed aside as disagreement with the data can be,
for it entails an unknown mechanism which determines
that range of the variable for which the distribution
must be modified as well as the unknown modification
itself, and leaves the researcher bereft of the argu-
ment that improved "experimental measurements" will
modify the situation in any agreeable way. It is much
easier to reconcile ill fitted observations, and their
consequences are normally much more local in nature.

Nevertheless it has been found desirable to modify
Zipf's Law in many ways to better fit the data. Mandel-
brot's modification, based on his theoretical consider-
ations, is

$$y = c(x - a)^{-s} \qquad (3.2)$$

where a is a small constant. Belonogov (11) found
that the distribution

$$y = e^{-c(x-1)^k} - e^{-cx^k} \qquad (3.3)$$

describes the rank-frequency structure of printed
commercial Russian. Good (12) is led to

$$y = c(x-a)^{-s(1 + by^{-1})}, \qquad (3.4)$$

with b a small constant; this form has (3.2) as a first
approximation (because b is small) and also is respon-
sive to Yule's criticism since the sum of the frequencies
is finite. It is derived by including in the effort
function (see below) a factor coresponding to the
effort required to incorporate words of large rank in
the inventory, and represents, in a certain sense,
part of the system 'overhead'. Unfortunately, (3.4)
is a complicated expression and Good's choice of
overhead factor is in no way uniquely determined.

Other authors have turned to functions that are appar-
ently quite different in order to more faithfully
describe their data. Houston and Wall (13) described
the distribution of term usage in manipulative indexes
using the lognormal distribution, eliciting from Fair-
thorne (14) the remark that, in his view, Wall (15)
(and presumably also Houston and Wall (13)) selected
the lognormal only because the data was well fit by
that distribution in the sense that the results plotted
as a straight line on lognormal probability graph
paper, but that they would also have done so on
ordinary logarithmic graph paper because "segments
of the tail of a Gaussian distribution are not readily
distinguished from segments of a hyperbolic distri-
bution"; by the latter he means the Zipf distribution.
Certainly this remark applies in principle to the figures
plotted on lognormal graph paper in the previous Chapter,
but we will see that it is significant only when the
variance of the lognormal distribution in question
is large.

Carroll (16) has discussed the statistical problems
associated with representation of the Standard Sample
of Present-Day Edited American English (17) by lognormal
distributions; there is in his work no hint of the
formerly used Zipf approximation.

We think that there are two conclusions that should
be drawn from this necessarily brief survey: first,
the Zipf-Mandelbrot distribution does not adequately
fit much data although it is well grounded in theory,
and second, it is often difficult to distinguish log-
normal approximations of data from Zipf approximations.
They suggest that an intimate relation may exist connect-
ing the Zipf and lognormal distributions, and, if this

be true, that a 'derivation' of the lognormal from elementary principles along the lines of Mandelbrot's arguments may be possible.

In order to show that these hopes are indeed justified, we will present a derivation of the Zipf-Mandelbrot Law following the usual argument, and in effect following Schrödinger (9), although the notation and terminology there is of course quite different.

Consider information 'states' S_1, $S_2,\ldots,S_x,\ldots$ constituting some inventory, such as the words of language as they occur in some large text corpus, or a large random collection of monograph titles or indexes, ordered in some convenient fashion. Let $\varepsilon(x)$ denote the 'effort' ('energy') required to utilize state S_x in an access system ('communication system'), and denote by $p(x)$ the probability of utilization of S_x in the inventory. Following Shannon (18), the expected amount of information per unit expected effort is proportional to

$$I = - \sum p(x) \log p(x) \; / \sum p(x) \varepsilon(x). \quad (3.5)$$

If the access system is such that the expected amount of information per unit effort is maximized, then the probabilities $p(x)$ cannot be unrelated to the effort function $\varepsilon(x)$; maximization of I subject to the necessary restraint

$$\sum p(x) = 1 \qquad\qquad (3.6)$$

will determine the form of $p(x)$ for given $\varepsilon(x)$. Now maximization of (3.5) subject to (3.6) is equivalent to maximization of

$$- \sum p(x) \log p(x) \qquad\qquad (3.7)$$

subject to (3.6) and the additional restraint that the total effort

$$\sum p(x) \varepsilon(x)$$

is constant as well. The mathematical method of Lagrange multipliers provides the solution to this extremal problem in the following way: since the total probability (3.6) and the total effort are constant, the function (3.7) and the function

$$H = -\sum p(x) \log p(x) \quad + \quad (1+a_0) \sum p(x)$$

$$+ \; a_1 \sum p(x) \epsilon(x) \qquad\qquad (3.8)$$

attain their maximum for the same functions $p(x)$ of $e(x)$, where a_0 and a_1 are arbitrary constants and the form $(1+a_0)$ has been chosen for later notational convenience. Now subject each $p(x)$ to small differentiable independent functional variations, all the while keeping x fixed; H will assume its maximum where the derivatives $\partial H/\partial p(x)$ all vanish. This yields the simultaneous conditions

$$0 = \partial H/\partial \; p(x) = -(1 + \log p(x)) +$$

$$(1+a_0) + a_1 \epsilon(x) \qquad (3.9)$$

which implies

$$\log p(x) = a_0 + a_1 \epsilon(x) \; . \qquad\qquad (3.10)$$

This is the fundamental relation connecting the effort function, which is presumed to be known, with the probability of occurrence of the state S_x. It remains to specify the effort function. Mandelbrot argued that, if the states S_x are words drawn from natural text and arranged in decreasing frequency of occurrence, then $\epsilon(x)$ is proportional to $\log(x-a)$ with a some small constant. This hypothesis immediately leads to (3.2) with $c = e^{a_0}$ and $s = -a_1$. In order that the distribution decrease with increasing x, a_1 must be negative. (If $a_1 = 0$, the distribution degenerates into the uniform distribution, which can only apply to a finite range of the variable x.)

The idea underlying Mandelbrot's choice of effort function is perhaps most simply illustrated by recalling the ordinary use of positional notation to represent positive integers. If b is an integer greater than 1 and n is any positive integer, then n has a unique representation of the form

$$n = a_N b^N + \ldots + a_k b^k + \ldots + a_1 b + a_0$$

with a_k integers less than b but not negative. For instance, if $b = 10$ and $n = 234$, then

$$234 = 2 \cdot 10^2 + 3 \cdot 10 + 4.$$

By means of such an expression n can be identified with the sequence of numbers $a_N a_{N-1} \ldots a_0$. If $b = 10$ this correspondence is the usual decimal expression for n, while if $b = 2$, it is the binary expression. Such an expression for n requires N symbols each of which is selected from an inventory of b symbols $(0,1,2,\ldots,b-1)$. Evidently

$$N + 1 \geq \log_b n \geq N,$$

so the number of places required to express n in base b is approximately $\log_b n$, and approximately $b \log_b n$ selections suffice to specify an integer lying between 0 and n.

By coding the information specified by the states S_x as integers, this argument can be made to apply to the S_x themselves, leading to the Zipf-Mandelbrot distribution for words if that is what the states represent.

It must be recognized that more is involved in the distribution of information states than the simple matter of minimal coding; Good's argument mentioned above attempts to account to some extent for the effort required to add a state to the inventory, i.e., to learn a rare word. Therefore the effort function may not have the form proposed by Mandelbrot except in the simplest of cases, and it becomes necessary to investigate the probable nature of substitutes for it. It might be argued that in general a multiple of $\log (x-a)$ will constitute a good first approximation to $\varepsilon(x)$. This, and equation (3.10), suggest the introduction of the variables

$$u = \log (x-a) \qquad\qquad (3.11)$$

and

$$f(u) = \log p(x) . \qquad\qquad (3.12)$$

so that (3.10) becomes

$$f(u) = a_0 + a_1 \varepsilon (\varepsilon^u + a) = \varepsilon^*(u), \qquad (3.13)$$

defining the function $\varepsilon^*(u)$ which is more convenient to work with.

Mandelbrot's assumption for the effort function is, in this notation, simply that

$$\varepsilon^*(u) = a_0 + a_1 u . \qquad (3.14)$$

We will assume that $\varepsilon^*(u)$ can be expanded in a Taylor series about the point $u = 0$, that is,

$$\varepsilon^*(u) = \sum_{k=0}^{\infty} a_k u^k . \qquad (3.15)$$

For small values of u, $\varepsilon^*(u)$ will be well approximated by the first two terms of (3.15) if a_1 is not zero, leading to the Zipf-Mandelbrot Law; if a better approximation is desired, more terms must be taken from the series expansion of $\varepsilon^*(u)$. Suppose for instance that an approximation accurate through terms quadratic in u is used:

$$\varepsilon^*(u) = a_0 + a_1 u + a_2 u^2 .$$

Using (3.11) through (3.13), we obtain

$$\log\ p(x) = a_0 + a_1 \log(x-a) + a_2 (\log(x-a))^2 ;$$

the right hand side can be written as

$$-\log\ (x-a) + a_2 \left[\left(\log(x-a)+(1+a_1)/2a_2\right)^2 \right.$$
$$\left. + \left(a_0/a_2 - \left(1+a_1\right)/2a_2{}^2\right) \right]$$

so

$$p(x) = c\ e^{\displaystyle -\frac{1}{2}\left|\frac{\log(x-a)+(1+a_1)/2a_2}{\sqrt{-1/2a_2}}\right|} \over (x-a) \qquad (3.16$$

$$= \frac{ce^{\displaystyle -\frac{1}{2}\left[\frac{\log(x-a)-m}{s}\right]^2}}{(x-a)}$$

with

$$c = e^{(4a_0 a_2 - (1+a_1)^2)/4a_2} \qquad (3.17)$$

which is the lognormal distribution with lognormal mean

$$m = (1 + a_1)/2a_2 \qquad (3.18)$$

and lognormal standard deviation

$$s = \sqrt{-1/2a_2} \qquad (3.19)$$

In other words, the parameters a_1 and a_2 which appear in the effort function $\varepsilon^*(u)$ are related to the parameters of the lognormal distribution defined by that effort function as follows:

$$a_1 = -(1 + \frac{m}{s^2}), \quad a_2 = -1/2s^2 , \qquad (3.20)$$

showing in particular that a_2 must be negative in order that the distribution correspond to a realizable system. Using these values for the constants appearing in the effort function yields

$$\varepsilon^*(u) = a_0 - (1 + m/s^2) u - u^2/2s^2 , \qquad (3.21)$$

which shows that Mandelbrot's hypothesis is warranted when s and m are large in such a way that the quotient m/s^2 remains finite. If s is not large, then necessarily the simple logarithmic effort hypothesis is inadequate.

This observation reconciles Fairthorne's remark, quoted above, but it has the further reaching consequence that the question of whether the Zipf or the lognormal provides a 'better' fit to given data is really meaningless from this point of view; the Zipf is a special case of the lognormal and can therefore never provide a better fit by the implied metric than the latter. Moreover, the larger s is, the easier it will be to confuse the two distributions, since the general form will more nearly approach its specialization as s increases.

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The derivation of the lognormal distribution given above is based on the measure, I, of information per unit effort defined by eq(3.5) which occurs in Mandelbrot's work and is also used by Good. It is, however, not the only reasonable measure of average effort. In fact, as W. E. Houchin has observed in a personal communication, the choice of

$$I^* = -\sum p(x) \left| \log p(x)/\varepsilon(x) \right| \quad ,$$

the expected value of the information per unit effort, in place of I (which is the expected value of the information per expected value of the effort) leads to the lognormal function in a more direct manner, free of the burdensome hypothesis that the effort function is quadratic in the logarithm of size or rank. For, arguing as before with I^* in place of I, leads to the maximization of

$$H^* = -\sum p(x) \left| \log p(x)/\varepsilon(x) \right| + a_1^* \sum p(x)$$

$$+ a_2^* \sum p(x)\varepsilon(x) \quad ,$$

and therefore to the equations

$$0 = \varepsilon(x)\partial H^*/\partial p(x) = -(1+ \log p(x)) + a_1^*\varepsilon(x) +$$

$$+ a_1^*\varepsilon(x)^2 \quad ,$$

with solution

$$\log p(x) = -1 + a_1^*\varepsilon(x) + a_2^*\varepsilon^2(x) \quad .$$

If the effort is a logarithmic function of x, $\varepsilon(x) = \log(x-a)$, then p(x) is the lognormal function with lognormal mean

$$m^* = (1+a_1^*)/2a_2^*$$

and lognormal standard deviation

$$s^* = \sqrt{-1/2a_2^*} \quad ;$$

these equations should be compared with eqs (3.18) and (3.19). If $m^* = 0$, then this lognormal function reduces to Zipf's original 'law' with exponent -1; if m^* and s^* are both large such that m^*/s^* is finite the general power function of Mandelbrot and Stevens results.

Recalling that $f(u) = \log p(x) = \varepsilon^*(u)$, (3.21) shows that the graph of $f(u)$ <u>vs.</u> u, that is, of $\log p(x)$ <u>vs.</u> $\log(x-a)$, will be a parabola if $p(x)$ is lognormally distributed, whereas it will be a straight line if $p(x)$ is distributed according to Zipf's Law. It is instructive to examine some examples.

Figure 3.2 displays the graph of the frequency of the most frequent ordered pairs of words drawn from English language text as a function of their rank, drawn on logarithmic graph paper. The data was derived from the corpus which constitutes the Standard Sample of Present-Day Edited American English (17). It approximates a straight line without any considerable evidence of global curvature thereby supporting the hypothesis that the effort function $\varepsilon^*(u)$ is linear and the consequent distribution Zipf. The arguments that are usually applied to justify the Zipf approximation for the distribution of frequency ranked single words would seem to apply equally well to this case, and therefore the arguments of Carroll (16) concerning the problems associated with the extraction of finite samples from theoretical distributions of lognormal type are also probably valid here, which helps to explain the bending of the curve in the direction of low frequencies for large ranks.

Next consider the distribution of the number of index entries in monographs, shown in Figure 3.3. The data is drawn from the Fondren Index Sample, which is described in detail in Chapter IV. Departure from linearity is clearly exhibited; this data is also shown in Figure 2.3, where it is plotted on lognormal probability paper with striking results which suggest that the points in Figure 3.3 should approximate a parabola. A portion of the parabola that fits this data is shown in the figure. The reader will notice certain peculiarities of the distribution of data points that are character- istic of this type of problem and lead to difficulties of estimation. First of all, small values of the independent variable--the number of index entries in this case---correspond to few data points if the data has been grouped for calculational convenience as these data have been. On the other hand, large values of the

Figure 3.2

Figure 3.3

INDEX ENTRY DISTRIBUTION
FROM
FONDREN INDEX SAMPLE

NUMBER OF MONOGRAPHS

NUMBER OF INDEX ENTRIES IN MONOGRAPH

75

independent variable correspond to many data points
even after grouping if the group intervals are of uni-
form size and for many of these the corresponding
frequencies will coincide, leading to vertical
'segments' such as appear over the '1', '2', and '3'
monograph markers. It is the geometric mean of these
values that is important if the distribution is in
fact lognormal.

More complex phenomena sometimes occur, and are perhaps
most easily initially analyzed by studying the nature
of the polynomial functions that approximate them, or
at least portions of them, in the variables u and f(u).
Consider, for example, the distribution of the number
of pages in a monograph, shown in Figure 3.4. The
data is drawn from the Fondren Sample (cp. Dolby, et.
al. (19)) and has been grouped. From the figure one
sees that monographs of fewer than 220 pages appear
to follow the Zipf distribution whereas longer monographs
have lengths that are well approximated by part of
a lognormal distribution since they correspond to data
points that fall nearly on part of a parabola as
shown in the figure; the small arrows indicate computed
values of points lying on the fitting parabola. We
have no satisfactory explanation for this curious
discontinuity in the effort function which describes
the distribution of lengths of monographs which maxi-
mizes information per unit effort. Extraneous factors,
perhaps related to the technology and economics of
printing, are probably responsible but we have thus
far been unable to isolate them. The reader should,
however, compare Figures 2.5 and 2.6 which show the
page distribution for unindexed and indexed monographs
in the Fondren Sample.

Now consider the problem of determining the parameters
of a lognormal distribution which represents given
data. Take the equation of the lognormal in the form

$$y = \frac{N}{s(x - a)\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\log(x-a) - m}{s}\right)^2} \qquad (3.22)$$

If the data consists of sample measurements $\{x_i\}$ such
that the sample frequency of occurrence of x_i is y_i,
then N is just the total number of measurements:

$$N = \Sigma \, y_i. \qquad (3.23)$$

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Figure 3.4

PAGE DISTRIBUTION
FROM
FONDREN SAMPLE

NUMBER OF PAGES IN MONOGRAPH

The values of a, m, and s are determined by introducing an auxiliary quantity related to the skewness of the sample distribution, to whose definition we now turn.

Some more terminology is necessary. Define the kth sample moment μ_k' by

$$\mu_k' = \frac{1}{N} \Sigma \, x_i^{\,k} y_i \; ;$$

μ_1' is the usual mean of the sample. If the sample moments are known, the central moments

$$\mu_k = \frac{1}{N} \; \Sigma (x_i - \mu_1')^k y_i$$

can be calculated. Expressions for the first four central moments in terms of the sample moments are useful when considering lognormal distributions. The first central moment μ_1 is evidently zero, since

$$\mu_1 = \frac{1}{N} \Sigma \; (x_i - \mu_1') y_i$$

$$= (1/N) \; \Sigma x_i y_i - (\mu_1'/N) \; \Sigma \, y_i$$

$$= \mu_1' - \mu_1' \; .$$

The next three central moments are given by the relations

$$\mu_2 = \mu_2' - (\mu_1')^2 \; ;$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3 \; ;$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^3 \; .$$

The positive square root of μ_2 is usually called the standard deviation and denoted by σ :

$$\sigma = \sqrt{\mu_2} \quad .$$

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Introduce the ratios

$$\beta_1 = \mu_3^2/\mu_2^3 \qquad\qquad (3.24)$$

and

$$\beta_2 = \mu_4/\mu_2^2 \; . \qquad\qquad (3.25)$$

β_1 is called the _skewness_ of the sample; it provides a simple measure of the departure from symmetry about its mean. The skewness of the normal distribution, as for all symmetrical distributions, is 0. β_2 is sometimes known as the _kurtosity_ of the sample distribution. If $\beta_2 < 3$, it is lower and flatter. The kurtosity of a normal distribution is 3.

In the literature, and unfortunately also in tables, other formulae are sometimes used to define quantities known as skewness and kurtosity. It is common, for instance, to find

$$\gamma_1 = \mu_3/\sigma^3 = \pm\sqrt{\beta_1}$$

called 'skewness' (note that the sign of γ_1 is the same as the sign of μ_3), and

$$\gamma_2 = \beta_2 - 3$$

is sometimes called 'kurtosity'. We follow Karl Pearson's usage, as found for instance in Ref. (20).

Skewness and kurtosity are of interest because they provide simple measures of the deviation of a sample distribution from the normal distribution and can be used to determine a family of distributions likely to provide an accurate and practical representation of data exhibiting skew variation; this procedure was introduced by Pearson (21). We will later have occasion to compare the lognormal representation of data occurring in information systems with representations by means of Pearson's distributions.

Skewness is of immediate interest here because the
three numbers μ_1, μ_2, γ_1 determine the lognormal distri-
bution that best fits given sample data. The unknown
parameters a, m, and s of (3.22) can be expressed by
means of an auxiliary quantity η which is the real
root of the equation

$$\eta^3 + 3\eta - \gamma_1 = 0 \qquad\qquad (3.27)$$

where γ_1 is the square root of the skewness (with the
correct sign) as defined in (3.25). It is easy to
see that there is in fact just one real root to (3.27),
for otherwise there must be three, so the graph of
the left side of (3.27) would have two turning points,
which means that the derivative of the left side would
have two real roots. But the derivative is $3\eta^2+3$,
whose roots are pure imaginary.

The unique real root of the cubic equation (3.27) is
readily and accurately approximated by using Newton's
method. First select some reasonable approximation
to the root; if a better choice is lacking, set

$$\eta_1 = \gamma_1/3$$

If η_k is the kth approximation to the root η, then the
next approximation is

$$\eta_{k+1} = \eta_k + (\gamma_1 - 3\eta_k - \eta_k^3)/(3\eta_k^2 + 3). \qquad (3.28)$$

For example, if $\eta_1 = 3$, then $\eta_1 = 3/3 = 1$, and

$$\eta_2 = 1 + (3 - 3 - 1)/(3 + 3) = 0.833333... ,$$

$$\eta_3 = 0.795556 ,$$

$$\eta_4 = 0.817973 ,$$

$$\eta_5 = 0.817732 = \eta_6 ;$$

therefore $\eta = 0.817732$ correct to six places.

Given a sufficiently accurate value of η, the parameters of the lognormal distribution (3.22) are (cf. Cramer, Ref. (22)):

$$a = \mu_1' - \sigma/\eta \ , \tag{3.29}$$

$$s = \{\log (1 + \eta^2) \}^{1/2}, \tag{3.30}$$

$$m = \log (\sigma/\eta) - \frac{1}{2}s^2 \ . \tag{3.31}$$

Recall that $\sigma = \sqrt{\mu_2}$ is the standard deviation.

If the parameters of a lognormal distribution are known, it is of course possible to calculate the kurtosity of that distribution; the result is expressible in terms of η as

$$\beta_2 = \eta^8 + 6\eta^6 + 15\eta^4 + 16\eta^2 + 3 \ , \tag{3.32}$$

and is of no particular value except that it permits one to sketch the graph of the skewness and kurtosity pairs (β_1,β_2) that can belong to lognormal distributions. Figure 3.5 shows such a graph. The skewness and kurtosity of a particular data sample determine a point in the β_1-β_2 plane; the farther this point is from the lognormal curve, the less likely is the hypothesis that the sample data is drawn from a lognormal distribution.

By applying the usual techniques of the differential calculus it is easily shown that the maximum value of the lognormal distribution (3.22) is attained when

$$x = a + e^{m-s^2} \ . \tag{3.33}$$

Some examples showing how these equations are to be applied to sample data may have some interest for the reader.

Consider first the distribution of the number of entries in a monograph index. For data drawn from the Fondren Index Sample, Figure 3.3 shows that this distribution apparently is lognormal. The following table, which summarizes the data in Table 4.3, groups the sample measurements according to intervals of 500 index entries and nominally associates them with the center value in each interval. This coarse grouping scheme

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Figure 3.5

COEFFICIENT OF SKEWNESS

VS

COEFFICIENT OF KURTOSIS

FOR LOGNORMAL FUNCTIONS

decreases the number of categories to the point where
the necessary calculations can be conveniently performed
on a desktop calculator.

Table 3.1

GROUPED NUMBER OF
INDEX ENTRIES FOR MONOGRAPHS

Nominal No. of Entries	Number of Indexes
250	326
750	214
1,250	76
1,750	36
2,250	17
2,750	9
3,250	6
3,750	8
4,250	2
4,750	6
5,250	1
5,750	1
6,250	2
6,750	1
7,250	1

One finds

$$N = 706$$

$$\mu_1' = 831.444759$$

$$\mu_2 = 878,281.765706, \quad \sigma = 937.166882$$

$$\mu_3 = 2,551,331,421.74$$

$$\mu_4 = 13,238,646,211,200$$

to twelve figures. Consequently

$$\gamma_1 = 3.0997$$

$$\beta_1 = 9.6080$$

$$\beta_2 = 17.1623 \ .$$

According to Figure 3.5, the sample value of β_1 corresponds to $\beta_2 \cong 24$ if the distribution is lognormal.
The disagreement is not serious in view of the effect of grouping the data and the small size of the sample; regarding the latter point, J. Carroll's remarks in Reference (16) are instructive. For the original ungrouped data, it turns out that $\mu_1' \cong 836$, so the effect of grouping the data is not entirely negligible.

These estimates imply $\eta = 0.837450$ correct to six figures; indeed, since γ_1 is nearly equal to 3, it is a good idea to select the solution to (3.27) previously calculated as an example as the starting value η_1 of the approximation procedure, leading to

$$\eta_1 = 0.817732,$$

$$\eta_2 = 0.837642,$$

$$\eta_3 = 0.837450 = \eta_4 = \eta \ .$$

Substitution in (3.29)-(3.31) produces the parameters of the lognormal:

$$a = -287.6 \ ,$$

$$s = 0.7284 \ ,$$

$$m = 6.755 \ ,$$

so

$$y = \frac{706}{0.7284(x+287.6)\sqrt{2\pi}} e^{-\frac{1}{2}\left\{\frac{\log(x+287.6)-6.755}{0.7284}\right\}^2}$$

Observe that

$$c^m \cong 858 \cong 874 \cong K^2.$$

According to these calculations, the (grouped) number
of index entries behaves as if approximately 288 entries
are 'missing' from indexes. To what extent this must
be attributed to the effect of grouping the data so
coarsely we have not attempted to determine except
to note that the intervals that were chosen will tend
to produce this type of qualitative effect because
most of the indexes represented in the first group
(those having fewer than 500 entries) contain more
than the 250 entries nominally ascribed to that cate-
gory, thus biasing the distribution toward low values.

The next example is a particularly useful pedagogical
illustration because the data are unusually regular
and occur in large number in a form convenient for
calculations, but it is of considerable independent
interest as well. Consider the distribution of
dictionary words according to the number of vowel
strings they contain. The number of vowel strings,
in the technical sense in which it is used here, is
a graphemic substitute for the phonemic notion of the
number of syllables contained in the spoken form of
a word; the precise definition we use is given in
Reference (23), but will not be necessary for our present
purposes since the intuitive correspondence with the
notion of syllable is sufficiently accurate. The
words under consideration are the 64,041 lexed words
of the <u>Shorter Oxford Dictionary</u> which contain at least
one vowel string. Figure 3.6 displays the data on
bi-logarithmic graph paper; the general parabolic
tendency is apparent. From the data given in Table 3.2
one readily calculates

$$N = 64{,}041 ,$$

$$\mu_1 = 2.6889 ,$$

$$\mu_2 = 1.1096 , \quad \sigma = 1.0534 ,$$

$$\mu_3 = 0.5027 ,$$

$$\mu_4 = 3.7011 ,$$

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Figure 3.6

SHORTER OXFORD DICTIONARY

NUMBER OF WORDS

10^5

10^4

10^3

10^2

10

1

2   3   4   5 6 7 8 9 10   {N + 4} VOWEL STRINGS

TABLE 3.2

DISTRIBUTION OF LEXED WORDS
FROM THE SHORTER OXFORD DICTIONARY
BY NUMBER OF VOWEL STRINGS

Number of Vowel Strings	Observed Number of Words	Calculated Number of Words
0	63	285
1	7,158	6,618
2	22,568	22,160
3	20,762	22,072
4	10,293	9,737
5	2,770	2,531
6	393	691
7	30	178
8	4	24

So

$$\gamma_1 = 0.4301 ,$$
$$\beta_1 = 0.1850 ,$$
$$\beta_2 = 3,0060 .$$

For a lognormal distribution, this value of skewness implies a kurtosity approximately equal to 3.3, which agrees reasonably well with the value computed from the sample. The calculated parameters of the lognormal fitting these data are

$$a = 4.7086 ,$$
$$s = 0.1407 , \qquad\qquad (3.34)$$
$$m = 1.9916 .$$

The maximum value of this lognormal distribution is $y \cong 25,000$ and occurs at $x \cong 2.46$. The rightmost column of Table 3.2 shows the number of words as a function of the number of vowel strings as calculated from the distribution defined by the parameters given in (3.34) above.

Some degree of caution must be exercised when one attempts to determine if data can be reasonably fitted by a lognormal function. If the entire range of the variable is not represented by the data due either to unfortunate grouping or absence of information, graphical representation of the data may be misleading. We will describe one way in which this can happen. Suppose that $\{(x,y)\}$ is a data sample exhibiting the frequency function of some variable x. Let $\{(x,Y)\}$ denote the cumulative frequency distribution defined by

$$Y(x) = \text{sum of } y(x_0) \text{ for } x_0 \geq x .$$

For instance, if y(x) denotes the number of individuals having an annual income of x dollars (more exactly, x+b dollars for some conveniently chosen small increment b), then Y(x) denotes the number of individuals with income <u>at least</u> x dollars. The latter cumulative distribution <u>is</u> exact in the sense that it presents the actual number of individuals belonging to the corresponding category of the sample, whereas the frequency distribution presents grouped data as a substitute for frequency density functions and therefore potentially introduces error into the sample data. For this reason it is often desirable to analyze cumulative sample distributions rather than the corresponding approximate frequency functions

Consider, therefore, a cumulative distribution $\{(x,Y)\}$ whose points fall close to a straight line when exhibited on log-log graph paper. In this event it is reasonable to conclude that $\log Y = a \log x + \log c$ so

$$Y = cx^a : \hspace{3cm} (3.35)$$

Y is a power function of **x**. The frequency function can be retrieved from (3.35) from the relation

$$y = dY/dx ;$$

we find

$$y = cax^{a-1} ,$$

so y is also a power function of x.

If the entire range of the variable x is not represented by the sample data, this procedure of determining the theoretical frequency function from its cumulative distribution by differentiation can be misleading. Figure 3.7

Figure 3.7

DISTRIBUTION OF INCOME

N = NUMBER WITH INCOME ≥ ORDINATE SHOWN

(N IN HUNDREDS FOR U.S.)

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displays the cumulative income distribution for Great
Britain in 1893-94 and for the United States in 1968
drawn on log-log graph paper. The data for the former
fall along a straight line which implies that it and
hence also the associated frequency function are power
functions. This is the famous 'law' of Vilfredo Pareto
(Ref. (6), vol.2, p. 304 et seq.). The leftmost part
of the corresponding distribution for the United States
also exhibits a generally linear trend but the rightmost
portion cannot be so construed at all. Several interpre-
tations of this anomoly are possible, including some that
are based on variations between the underlying economic
and social structures of the two nations during the two
time periods surveyed, but it is possible to account
for the apparent contradiction by examining the extent
to which the sample data represents the entire range of
variation.

The United States data refers to income of any size
reported on tax returns, of which more than 73 million were
tallied. The data used by Pareto refers only to incomes
greater than 150 pounds sterling per year, of which 400,648
were reported. The number of inhabitants per income re-
ported was nearly 3 for the United States in 1968;
using this figure, we see that if Pareto's data includes
essentially all incomes, the population of Great Britain
in 1893-94 should have been about 1.2 million. It was
in fact perhaps greater than 8 million, which suggests
that there were possibly more than two million people
in Great Britain in those years having a positive income
less than 150 pounds per year. The reader should not
interpret this estimate as anything but a very crude
indication of the number of incomes that were probably
overlooked in the data sample. Now apply this estimate
to extend the graph for Great Britain in Figure 3.7;
the extension must turn downward when the total number
of incomes exceeds about 2.4 million, so the extended
income curve will have the same general appearance as
that for the United States.

As has already been remarked, the cumulative distribution
of income function for the United States certainly is
not a power function and therefore the corresponding
frequency functior cannot be either. By plotting the
grouped frequency data published by the Internal Revenue
Service on lognormal probability graph paper, it can be
seen that the frequency function of income distribution
can be approximated throughout its entire range by a
lognormal function with the parameter a of eq (3.22)

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approximately equal to $4,000. It is likely that the
income distribution for Great Britain used by Pareto
can also be approximated by a lognormal function, but
it is necessary to have an accurate estimate of the total
number of incomes less than 150 pounds per year before
one can calculate the cumulative fractions necessary
for the employment of lognormal probability graph paper
or the methods for estimating the parameter values from
sample data given earlier in this Chapter.

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CHAPTER IV

THE STRUCTURE OF
BACK OF THE BOOK INDEXES

THE STRUCTURE OF

BACK OF THE BOOK INDEXES

Book indexes are among the most common and most
ancient access mechanisms, although they have not
always been loved. Glanville, in Vanity of Dogmatizing,
said:

> Methinks 'tis a pitiful piece of knowledge
> that can be learnt from an index, and a poor
> ambition to be rich in the inventory of another's
> treasure,

and more recently T. E. Lawrence wrote:

> ...half-way through the labor of an index
> to this book I recalled the practice of my
> ten years' study of history; and realized
> I had never used the index of a book fit
> to read.

However, as an unnamed contributor to a recent edition
of the Encyclopedia Britannica put it,

> (It has) become almost a sine qua non that any
> good book must have its own index.

Indeed, as we shall see below, more than one-third of
all non-serial items in the shelf list of a medium
size university library do contain an index, and it
seems as if the back of the book index is not only
here to stay but is in the process of spawning a genus
of related tools for indicating "the position of
information on any given subject".

The object of this chapter is to study indexes to
books in order to determine what structure, if any, they
possess. It is not surprising that indexes* exhibit
great variability in size, content, and utility, which
makes it difficult to assess their nature in general from
an examination of one or several exemplars. We have
elected to study indexes in three ways.

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*Throughout this chapter 'index' will only refer to
back-of-the-book indexes.

The first and most reliable way is based on the selection
of a random sample of book indexes. Such a sample has
been assembled by extraction of the indexes from all
monographs represented in a random sample of the shelf
list of a medium size university library; it consists
of approximately six hundred thousand index terms
spread throughout some 700 books, and will be described
in what follows:

The second means of studying indexes is concerned with
the structure exhibited by each index separately.
Information of this sort cannot be obtained from statis-
tical agglomerations; rather it demands that indexes
be considered in detail and the resulting structures,
if any are found, compared for a sample of indexes.

A book index directs the user to the location of
specified information in the book to which it refers.
Should the book in question not contain any indexed
information about the subject of interest, the inquirer
is left to continue his search in the indexes of other
unspecified books. There are, of course, several indirect
methods for deciding how the next book in the search
process should be selected, utilizing information con-
tained in the bibliographies or the linear shelf list
order determined by a subject classification scheme
such as that of the Library of Congress, but none of
these have the virtue of immediacy nor of completeness.
Our third means of studying indexes is based on a
cumulative index to 80 books in the field of statistics.
It appears to offer attractive efficiencies in the
information search process while it provides a view of
the overall structure of the field itself.

The Fondren Index Sample is a random sample of 668
monograph shelf list cards corresponding to indexed
books. Multiple volumes catalogued on one shelf list
card increase the sample somewhat so that a total of
706 indexes are represented.

The Fondren Index Sample is a subsample of the Fondren
Sample, which is a random sample of cards drawn from
the shelf list of the Fondren Library at Rice University.
The Fondren Sample is described in some detail in
Reference [1]. Analyses of the sample may be expected
to accurately reflect the structure of library collec-
tions to the extent that they are similar to the
Fondren collection; in particular, the archival
collections of medium size university libraries are
probably generally similar although certain special fields

may be more or less well represented. For instance, the Fondren collection is particularly weak in law, medicine, and Russian language and literature, and strong in chemistry. These differences are unlikely to play a significant role in determining the reliability of the sample for studying index structure since indexes are relatively insensitive to the nature of the subject material to which they refer; the gross category differences, as between science and fine arts, are, as will be shown below, substantial, but the Fondren collection encompasses adequate representation in each of such broad categories.

There are special problems associated with the analysis of complex data drawn from any sampling process. The index sample is no exception. Some of the sample indexes have a format so unusual as to make them incomparable with the average index; a small number were written in non-Roman alphabets so we were unable to correctly identify the structural features of interest. Because the fraction of anomolous indexes was small, it was decided to delete them from the index sample for this initial study.

This decision was bolstered by another complication; not all of the books represented by the original random sample could be located for the present study, which took place about two years after the original selection of shelf list cards. The number of unlocatable items was 33, approximately 1.7% of the Fondren Sample; this is the effective rate of loss for the two year period in the sense that the usual mechanisms for tracking items not present on the shelf in their proper location were applied without success for these items, noting that just prior to the selection of the sample the shelf list had been checked against the shelf and weeded. This suggests that slightly less than 1% of the monograph archive is lost each year.

If all 33 unlocatable items had had indexes, they would have constituted nearly 4% of the index sample; items excluded for special reasons such as language or format incompatibility totalled 22. Therefore, not more than 7.5% and more likely not more than 4.5% of the indexed volumes in the Fondren Sample have been excluded from the index sample. With this preliminary in mind we can now turn to the consideration of the index sample.

First observe that not all monographs are candidates for indexing; we have found no Library of Congress class "A" items in the sample which contain an index,

and therefore class "A" is excluded from all further
considerations. Similarly, neither maps nor musical
scores are indexible in the "back of the book" sense,
so they too are excluded. Excluding these items and
all serial publications, one finds that there are 1,830
relevant items in the Fondren sample. Of these, 668
have indexes; thus we find that 37% of the monographs
in the Fondren sample contain indexes.

As previously noted, the 668 LC cards lead to a total
of 706 volumes with indexes. The distribution of these
706 volumes by LC class is shown in Table 4.1 together
with the fraction that is indexed for each class.
This fraction runs from a low of 0.18 for N (Fine Arts)
and P (Language) to a high of 0.61 for Q (Science)
and 0.67 for Naval Science.

Table 4.1 also provides the mean number of index
entries per book indexed. The grand mean for the collec-
tion is 836 index entries per book, with the class means
varying from a high of 1,391 entries per book for class F
(U.S. Local History) to a low of 614 for class J
(Political Science).

The product of these two measures provides an average
measure of the amount of access per book in the collec-
tion and in each of its subsets. This distribution is
shown separately in Table 4.2. This list breaks
rather naturally into three subsets of nearly the same
size. The first seven categories (classes F, G, V,
K, D, E, and Q) would seem to share the property that
they are all primarily concerned with careful descrip-
tion of the world as it is and as it has been. The
middle group (classes H, C, R, T, Z, L, and J) is
primarily devoted to man's effort to cope with the
environment described so carefully in the first group.
The lowest group appears a bit anomolous in that it
contains the core of the arts: music, philosophy,
religion, language, literature, and the fine arts as
well as the more mundane but ever present categories
of war and agriculture. Although we should not like
to make too much of this particular arrangement of
the LC classes, Table 4.2 does provide an interesting
example of the insight one gains into the use of the system
of literary stores by rather elementary counting procedures.

The index sample consists of a total of 590,329 index
entries spread across the 706 indexes. Table 4.3 lists
the number of indexes as a function of the number of
entries they contain, grouped by hundreds of index
entries. Figure 4.1 exhibits the lognormality by showing
the data of Table 4.3 plotted on lognormal paper. The
standard deviation on the log scale is 0.442 which is at
the upper end of the range for log-length distributions
given in Chapter II.

Table 4.1

FONDREN SAMPLE: FRACTION OF SAMPLE ITEMS CONTAINING AN INDEX, BY LC LETTER CLASS

Class	Mean Number of Entries per Index	Fraction Indexed (rounded)	Fraction Class is of Fondren Sample	Short Class Name
B	667	.31	.100	Philosophy-Religion
C	690	.53	.009	History-Auxiliary Sciences
D	1,102	.51	.095	History & Topography (except America)
E	1,062	.49	.040	American (General) & U.S. (General)
F	1,391	.46	.027	United States (Local) & America (ex. U.S.)
G	1,264	.50	.011	Geography-Anthropology
H	697	.54	.104	Social Sciences
J	614	.46	.023	Political Science
K	1,375	.43	.004	Law
L	620	.49	.038	Education
M	915	.25	.015	Music
N	615	.18	.033	Fine Arts
P	714	.18	.300	Language & Literature
Q	850	.61	.093	Science
R	716	.50	.010	Medicine
S	638	.20	.006	Agriculture-Plant & Animal Husbandry
T	707	.47	.032	Technology
U	840	.22	.010	Military Science
V	934	.67	.005	Naval Science
Z	1,328	.24	.023	Bibliography & Library Science

Total relevant items in Fondren Sample = 1823

Number of these items indexed   = 668

Fraction indexed   = 668/1830   = 0.37

Table 4.2

INDEX ACCESS BY LC CLASS

LC Class	Mean No. Index Entries per Book	Short Class Name
F	640	U. S. Local History
G	632	Geography
V	626	Naval Science
K	591	Law
D	562	World History
E	520	U. S. History
Q	519	Science
H	376	Social Science
C	366	Auxiliary Sciences (History)
R	358	Medicine
T	332	Technology
Z	319	Library Science
L	304	Education
J	282	Political Science
M	229	Music
B	207	Philosophy-Religion
V	185	Military Science
P	129	Language Literature
S	128	Agriculture
N	111	Fine Arts

Table 4.3

FREQUENCY OF INDEX ENTRIES FOR ITEMS
IN THE FONDREN INDEX SAMPLE

Number of Index Entries	Number of Indexes	Cumulative Number of Indexes	Cumulative Fraction of Indexes
0 - 99	16	16	.023
100 - 199	77	93	.132
200 - 299	83	176	.249
300 - 399	80	256	.362
400 - 499	70	326	.462
500 - 599	62	388	.549
600 - 699	46	434	.615
700 - 799	37	471	.667
800 - 899	39	510	.722
900 - 999	30	540	.765
1000 - 1099	17	557	.789
1100 - 1199	24	581	.823
1200 - 1299	13	594	.841
1300 - 1399	14	608	.861
1400 - 1499	8	616	.872
1500 - 1599	2	618	.875
1600 - 1699	13	631	.893
1700 - 1799	7	638	.903
1800 - 1899	7	645	.913
1900 - 1999	7	652	.923
2000 - 2099	7	659	.933
2100 - 2199	3	662	.937
2200 - 2299	5	667	.944
2300 - 2399	1	668	.946
2400 - 2499	1	669	.947

Table 4.3
(Continued)

2500 – 2599	2	671	.950
2600 – 2699	3	674	.955
2700 – 2799	3	677	.959
2800 – 2899	1	678	.960

3000 – 3099	3	681	.964
3100 – 3199	2	683	.967
3300 – 3399	1	684	.969
3500 – 3599	1	685	.970
3700 – 3799	2	687	.973
3800 – 3899	3	690	.977
3900 – 3999	2	692	.980
4000 – 4099	1	693	.981
4200 – 4299	1	694	.983
4700 – 4799	3	697	.987
4900 – 4999	3	700	.991
5100 – 5199	1	701	.993
5900 – 5999	1	702	.994
6200 – 6299	2	704	.997
6700 – 6799	1	705	.998
7000 – 7099	1	706	1.000

Figure 4.1

Distribution of Index Length
by Number of Index
Entries
Fondren Index Sample

A distinction should be made between the number of index entries in an index and the number of locations to which these entries refer. The former quantity is the number of distinct word sequences appearing in an index, and is an absolute measure of index size which is independent of the details of format and page composition; the latter is usually the number of page locations referred to in an index, which clearly depends on the size of the page. In the Fondren sample of indexed books there are, on the average, 1.8 page locations per index entry. Thus, the 836 (average) distinct entries refer, on the average to 1,505 text locations. As there are on the average 341.5 pages per indexed book, there are 4.4 indexed text locations per page. Roughly speaking, this means that there is one index page location for each five sentences of text.

The aggregate size of the index as printed can be determined by estimating the average number of characters per entry and multiplying by the average number of entries. A preliminary estimate of the average number of characters was obtained by counting the entries in the cumulative index to statistical books (discussed at greater length in Chapter VI) as the format of the material is in particularly nice form for counting purposes. This estimate shows that the entries are about 25.47 characters in length exclusive of page location information. If, as in Chapter I, this is augmented by 4 characters per entry to include the typical page location reference information, then the average index of 836 entries consists of 24,637 characters and therefore the ratio of indexed book size to index size is about 33.27 to 1.

These global statistics provide a direct measure of the proportion of the monograph collection that is devoted to what might be called "self access". The aggreement of the access ratio (of about 30 to 1) with other access ratios developed in Chapter II helps to solidify the foundations of the level structured access model. Given the difficulty of assessing the quality of indexing (see (2) and the references therein) these statistics also provide the foundation of a basis for comparing various indexing procedures, particularly for comparing algorithmically derived indexes to manual indexes. The fundamental regularities of the length measures discussed here suggest that an algorithmically prepared index must at least be of the correct overall size to be of any use at all.

The find structure of the individual indexes can presumably shed more light on the situation. For these purposes, we have selected a random sub-sample of 28 indexes from the main Fondren Index Sample. For each of these indexes we have determined the distribution of the number of entries with one, two, three...page locations per entry. This distribution is comparable to the "frequency of frequencies" problem discussed extensively by Zipf, Bradford, Mandelbrot, et al (see Chapter III). Were the index an extractive index (i.e., one that is derived by extracting sequences of words from the text and inserting these sequences without change in the index) and were the page locations explicitly tied to the position on the page so that multiple occurrences of the entry on a single page would occur multiply in the index, then it might be anticipated that the text location distribution of index entries would be Zipf-Mandelbrot distribution which would arise from the phrases which are the index entries in the same way as the usual Zipf distribution arises from text word occurrences.

However, indexing practice normally requires a set of sophisticated transformations from the running text to the index and also reduces multiple entries on a page to a single page location. Further, not all "phrases" are indexed and it would appear that those which are left out are among both the most frequently occurring and least frequently occurring. Nevertheless, it seems reasonable to approach the problem at the first order of approximation by assuming a model of the Zipf-Bradford-Mandelbrot type; i.e., by examining the form of the distribution on log-log graph paper. This has been done for all 35 of the sample indexes, all 28 of which are presented here (Figures 4.2). (The remaining graphs appear in Appendix II.) The plots are given in the converse form to that used by Zipf in order to provide the converse form to that used by Zipf in order to provide stability (see Kendall (3)). Thus the largest point on the graph represents the number of index entries with single page references rather than the number of page references for the most frequently referenced item.

Two graphs shown are typical for the sample as a whole. In almost every case a straight line provides a reasonable approximation, with slopes ranging from roughly -1.1 to 05.5. Thus the Zipf-Mandelbrot approximation holds well for index location frequency distributions. The importance of the slope as a parameter of index measurement can be seen by recalling the Mandelbrot formulation which maximized the expected information per unit effort; the reader may find it useful to compare e.g. (3.5) ff:

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Figure 4.2

Number of Index Entries vs. Number of Page References

PN2598.k4 b6 1931

slope = -1.66

NUMBER OF INDEX ENTRIES

NUMBER OF PAGE REFERENCES

ND553.D774 T6 1965

Slope = -2.00

NUMBER OF INDEX ENTRIES

NUMBER OF PAGE REFERENCES

$$I = \frac{- \Sigma\ p(x)\ \log\ n}{\Sigma\ p(x)\ \log\ x} \qquad\qquad (4.1)$$

The function that maximizes this ratio is the Zipf-Mandelbrot distribution:

$$p(x) = c\ x^{-s} \qquad\qquad (4.2)$$

Substitution of (4.2) into (4.1) yields

$$I = - \frac{\Sigma\ (\log\ c - s\ \log\ x)\ cx^{-s}}{\Sigma\ cx^{-s}\ \log\ x} = \frac{\Sigma x^{-s}\ \log\ \Sigma x^{-s}}{\Sigma x^{-s}\ \log\ x} \quad (4.3)$$

where all logarithms are to the base e and the summations extend   from 1 to the maximum number of page references per index entries.

For s greater than one, the summations all converge to functions of the Riemann zeta function as the maximum number of page references per index entry increases. Hence, with the sums running overall positive integers,

$$I = s - \frac{\zeta(s)\ \log\ \zeta(s)}{\zeta'(s)} \qquad\qquad (4.4)$$

An s increases, the ratio on the right, in turn, converges to $(\log\ 2)^{-1} = 1.443$ so that a first order approximation to Mandelbrot information for the Zipf-Mandelbrot form is given by

$$I = s + 1.443$$

For s greater than or equal to 3, the error is less than 10%.   In other words, to a first order approximation, Mandelbrot's measure of information per effort is directly proportional to s, the negative value of the slope of the approximating straight line on log-log paper.

For data that perfectly fits the Zipf-Mandelbrot model, the parameter s can be determined from the relation:

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$$s = \frac{\log \text{ (number of references with single page locations)}}{\log \text{ (number of page locations of most popular index entry)}}$$

clearly, the greater the number of single page location
index entries and the fewer the number of multiple
page location index entries, the greater the estimate
of s and hence the greater the amount of information
per effort under Mandelbrot's definition.   In the extreme
case, where each index entry refers to one, and only
one, page location, Mandelbrot information is infinite.
Although we have found so such indexes in the Fondren
sample, it is well to note that dictionaries take this
form: each main entry occurs once and the referent
information is conveniently packaged with the main
entry itself rather than through a page location to some
other source.

The values of s for each index in the subsample are
listed in Table 4.4 in decreasing order of s.   Earlier
in this chapter we organized the various monographs
by LC class and then by total number of entries per
monograph.   Under this measure the LC classes fell into
three disjoint sets corresponding roughly to the descrip-
tive materials, the technique materials, and the arts.
The average slope for each of these three groups are
respectively, 2.83, 2.37, and 2.79.   The differences
between the means are not only insignificant statistically;
they do not even provide a corresponding ordering, were
they significant.   Thus the slope (and hence Mandelbrot's
measure of average information per average effort)
provides an independent measure of the index.

The 28 values of s are plotted in Figure 4.3 on log-
normal paper.   The distribution of values is reasonably
approximated by a straight line as might be expected
since as we have now shown, s is a normalized measure
of information.

However, except for specialized indexes such as diction-
aries, multiply occuring entries do occur, thus depressing
the information ratio.   For the sample plotted in Figure
4.3, the average value of s is 2.66, quite close to the
natural constant, e, which is 2.718.   As these multiply
occurring entries do reduce the information ratio by
increasing the effort required, it is appropriate to
inquire as to what role they play in the index.

Some hint as to the nature of this phenomenon can be
obtained by examining the role of the multiply occurring
entries in the context that Zipf first studied them;

Table 4.4

ZIPF-MANDELBROT EXPONENT
FOR INDEX LOCATION DISTRIBUTION

LC Number	S
PT7244	5.47
QD9	4.43
HB199	4.33
BV2532	3.81
DA690	3.54
TK153	3.53
E741	3.39
Q391	3.20
QA303	3.06
E178	2.81
QL703	2.71
RM721	2.71
BF181	2.69
DF521	2.64
Z5782	2.49
ND553	2.26
HM66	2.15
F864	2.08
PR2831	1.96
LB875	1.94
LC191	1.93
HF2046	1.86
D443	1.81
HD20	1.71
PR5588	1.69
PN2598	1.67
DS423	1.43
JA84	1.09

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Figure 4.3

Distribution of Zipf-Mandelbrot Slopes
Subsample of the Fondren Index
Sample

s - Zipf-Mandelbrot Slope

PROBITS

PERCENTAGE

in natural language itself. Even a cursory examination
of a frequency ordered word list such as those prepared
by Thorndike and Lorge (4) and Kucera, et al, (5)
is sufficient to show that the most frequently occurring
entries are the structure words (i.e. words with parts
of speech other than noun, verb, adjective, and adverb).
Such words provide the structure in which the information
is embedded, but do not, at least in the broad sense,
contain information themselves. Except for the rare
case (e.g. in the use of certain prepositions in mathe-
matical treatises) such words almost never occur in
first position in an index entry.

In this context, it seems natural to suggest that the
index entries that occur with many page locations play
a fundamentally different role from those that refer
only to one or a few page locations. Roughly speaking,
we might say that the multiply occurring entries carry
the semantic structure in much the same way that the
multiply occurring words carry the syntactic structure.
Suppose, for instance, that the term California appears
in an index with, say, 15 page locations. It would
seem reasonable to conclude, even with no other informa-
tion about the accompanying text, that the text is
very much concerned with California in a global manner.
Reference to each of the various page locations would
presumably uncover a variety of bits of information about
California and in this particular sense, we could say
that California was one of the "subjects" discussed
in the book. If on the other hand, we were to find
another book, say on population statistics, whose index
contained a single page location for California, it
would seem appropriate to conclude that California was
one of many items discussed in the text rather than a
main subject of the text.

In short, if one is interested in "population statistics
for the state of California" one can either go to a book
on population statistics and look in the index for
California, or one can go to a book on California and
look in the index for population statistics. For obvious
reasons both types of information packaging exist and access
to the packaged information is generally, though not
always, provided both ways: by subject to allow the
user to get to the proper book, and by index entry to
allow the user to obtain the specific fact once he
has gotten to the proper book.

The multiply occurring entries thus provide a sort of
transition from the "specific fact" aspect of the problem
to the "general subject" aspect of the problem. They
provide the basis for an algorithmic identification of

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the semantic structure in the same way that the structure
words provide a basis for the algorithmic identification
of the author's syntactic style. (See Mostellor and
Wallace (6))

For both the word frequency distribution and the index
page location distributions, there is no clear break
between the set of frequently occurring items and the
set of non-frequently occurring items. However, the
previously developed arguments on the access level
structure provide a technique for establishing break
points in the distribution: the set of most frequently
occurring entries can be defined as 1/900th of the whole
set of entries. This has been done for the subsample
of indexes from the Fondren sample. The results are
tabulated together with the LC class, the LC subject
headings, and the title in Table 4.5.

Looking first at the subject heading and title information
in Table 4.5, it is clear that approximately two-thirds
of the subject headings are direct transformations (through
the subject heading authority list) of the title
information. This observation, of course, sheds con-
siderable insight into the discussion of the utility
of permuted title indexes: anything as cheap as a permuted
title listing that can supply in the order of two-thirds
of the subject heading information automatically is
clearly useful. At the same time a device that misses
one-third of the potential information is clearly not
sufficient.

In this context the role of the multiply occurring index
entries becomes more obvious: most of LC subject headings
that are not derivable from the title information
are derivable from the multiply occurring index entries
either directly (e.g. Andalusite, U.S.A. vs. Andalusite)
or at a higher level of synthesis (e.g. gaseous discharge
tube + ultra violet light + reaction, reactors vs.
electrical apparatus and appliances). At this stage it is
not necessary to re-open the much discussed question of
whether classification of documents can be obtained
economically through purely algorithmic processes;
other simpler problems must be solved first (e.g. the
automatic derivation of the index itself). However, it
is essential to obtain a clearer understanding of how
the various access devices already in operation interact
with one another. The preliminary results derived from
Table 4.5 make it clear that there is a direct relation
between the LC subject headings, the monograph titles,
and the multiply occurring index entries. The utility
of title derived indexes is manifest by their present use
and persistence. It remains to determine the utility of

Table 4.5

COMPARISON OF HIGH-FREQUENCY INDEX ENTRIES
WITH LC SUBJECT HEADINGS & TITLES

LC Class	1/900th Entires	LC Subject Headings	Title
BF181	1. Marston, W. M. (Author) 2. Freud, Sigmund	1. Psychology, Physiological	Integrative Psychology
BV2532	1. Fallen, The	None	The History of the Foreign Missionary Society
D443	1. Great Britain mentioned	1. Europe-Politics-1914	Ten Years of War & Peace
DA690	1. Sackville, Lady Margaret (afterwards Countess of Thanet) mentioned in Lady Anne Clifford's Diary	1. Knole Park, Sevenoaks, Engl. 2. Sackville Family	Knole and the Sackvilles
DF521	1. Churches: in Constantinople 2. Frescoes	1. Byzantine Empire-Civilization	Byzantium
DS423	1. Krsna 2. Siva 3. "Bhagavad-Gita" 4. Visnu 5. Brahman 6. Guru(s)	1. India-Civilization	The Cultural Heritage of India
E178	1. Beard, Charles A. & Mary 2. Jefferson, Thomas 3. Turner, Frederick Jackson	1. U.S.-Hist.-Addresses, Essays, lectures 2. U.S.-Hist.-Historiography	Understanding the American Past

Table 4.5 (Continued)

LC Class	1/900th Entries	LC Subject Headings	Title
E741	1. Prices: agricultural 2. Foreign Relations: Anglo-American 3. Federal Income Tax: individual 4. Tax: individual income 5. Farmers, income of 6. Legislation: agricultural 7. Agricultural, legislation for 8. Railroads: rates of	1. U.S.-Hist.-20th cent.	American Epoch
F864	1. Mass	1. Ansa (sic!) Juan Bautista de 2. California-descr. and travel 3. San Francisco-Hist.	Anza's California Expedition
HB199	1. Terborgh, Gene 2. Breakeven Charts, Examples of	1. Economics	Engineering Economy
HD20	1. Charts on simulated business results	1. Operations Research 2. Industrial Management-Research	Operations Research for Industrial Management
HF2046	1. Chamberlain, J.	1. Free trade and protection --Free Trade. 2. Tariff--Gt. Brit.	The Return to Protection
HM66	1. Trade Unions	1. Sociology	Social Theory
JA84	1. Economy	1. Political Science-Hist.-Russia.	Russian Political Thought

Table 4.5 (Continued)

LC Class	1/900th Entries	LC Subject Headings	Title
LB875	1. America	1. Education 2. Literature-Study and Teaching	Two Views of Education
LC191	1. Children, Disease of	none	Education and Social Progress
ND553	1. "Bride Stripped Bare By Her Bachelors, Even, The" (Ducamp)	1. Duchamp, Marcel, 1887- 2. Cage, John 3. Rauschenberg, Robert, 1925- 4. Tinguely, Jean, 1925-	The Bride and the Bachelors
PN2598	1. Butler, Pierce	None	Fanny Kemble
PR2831	1. Greg, Walter	1. Shakespeare, William. Romeo and Juliet 2. Shakespeare, William-Bibl.-Quartos	The Bad Quarto of Romeo and Juliet
PR5588	1. Keats, John	None	Theme and Symbol in Tennyson's Poems to 1850
PT7244	1. Bjark: Bjarkamal, anon.	1. Scalds and Scaldic Poetry 2. Icelandic and Old Norse Poetry	Den Norsk-Islandska Skaldediktningen
QA303	1. Cauchy, A. 2. Euler	1. Calculus	Vorlesungen Uber Differential und Integralrechnung

Table 4.5 (Continued)

LC Class	1/900th Entries	LC Subject Headings	Title
QD9	1. Gregory	1. Chemistry-Bibl. 2. Reference Books	Library Guide for the Chemist
QE391	1. Researves, India 2. Andalusite, U.S.A.	1. Sillimanite 2. Andalusite 3. Cyanite	Sillimanite
QL703	1. Carnivore 2. Bat(s)	1. Mammals	Principles in Mammalogy
RM721	1. Muscle Contraction	1. Gymnastics, Medical	Therapeutic Exercise
TK153	1. Tube, gaseous-discharge 2. Hertz 3. Light, ultra-violet 4. Maxwell 5. Reaction, Reactors 6. Valence electrons	1. Electrical apparatus and appliances 2. Electrons	Electrons at Work
Z5782	1. Passion 2. Comedy 3. Latin 4. Staging	1. Drama, Medieval-Bibl.	Bibliography of Medieval Drama

index entries, over and above their obvious utility in providing access to a book's contents, once the book is in hand. This question will underlie much of the discussion in the next two chapters.

Before turning to this question, however, it is useful to shed some light on how the indexer controls the multiplicities in the index and hence the value of s and the shape of the particular entries that will receive the highest numbers of page locations. Obviously, this can be done in several ways involving such delicate questions as the determination of how the indexer decides whether a particular word, or sequence of words, on a particular page should rate an entry in the index. At a simpler level, the indexer has the opportunity to reduce multiplicities by increasing the length of the entry. Thus in a work on history, the indexer can either provide a single entry for war, with a large number of multiplicities, or he can break this same set of entries down into subsets involving particular wars such as civil war, world war, etc.

That this mechanism is in fact used is easy to demonstrate. Table 4.6 provides the frequency distribution for the 27,188 index entries in a uniform random subsample of 35 indexes in the Fondren sample by word length. As might be expected, the distribution can be reasonably approximated by a log-normal distribution as shown in Figure 4.4. The arithmetic mean of this distribution is 3.68 words per index entry. Only 13.5% of the entries are one-word entries. This is somewhat larger than the 9.1% found in a smaller sample of indexes to statistical books studied by Dolby (7) but still provides strong support for the hypothesis advanced in (7) that the great bulk of the entries in back-of-the-book indexes are multi-word entries.

This observation has considerable significance for the design of automatic indexing procedures. If one-word entries constitute only 13.5% of the total index, it seems unlikely that detailed frequency studies of words will provide much insight into the problem of deriving index entries automatically. In some of the earliest work on this subject, Luhn (8) attempted to derive indexes from word frequency counts, with limited success. More recently, Damerau (9) established a procedure for deriving coordinate index terms (to be used later via machine searches) based on word frequency counts. Bloomfield's (2) study of Damerau's procedure makes it clear that coordination of the single terms derived by Damerau rarely leads to an index entry derived by humans for the same material. As we shall show in the next chapter, there is more to be gained by deliberately suppressing the one-word entries, rather than by attempting to emphasize them.

Table 4.6

Distribution of Index Entries by
Word Length - Subsample of
The Fondren Index Sample

Number of Words	Number of Entries	Cumulative Number	Cumulative Percentage
1	3673	3673	13.51
2	6563	1023ξ	37.65
3	4817	15053	55.37
4	3905	18958	69.73
5	2839	21797	80.17
6	1969	23766	87.41
7	1243	25009	91.99
8	801	25810	94.93
9	516	26326	96.83
10	281	26607	97.86
>10	581	27188	100.00

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Figure 4.4

Distribution of Index Entries
by Word Length
Subsample of the Fondren Index
Sample

The observation that index entries are usually one-word
entries also has some impact on a variety of questions
involved with the use of indexes in agglomerated form.
This will be discussed at some length in Chapter VI.

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CHAPTER V

ALGORITHMIC TEXT INDEXING

ALGORITHMIC TEXT INDEXING

An index increases access to a particular corpus of
information. Until recent times most indexes followed
the text material in certain types of books. Although
this may still be true today, the emphasis of research
into the nature of indexing has shifted to indexes
of other types of corpora, such as the permuted title
index and its variants and the citation index,
which index collections of document titles rather than the
text of the documents. Indeed current information
retrieval efforts appear to exclude consideration of
back-of-the-book indexes. For instance, Salton (1)
offers a brief discussion of term-oriented, or derived
indexes, of which the back-of-the-book indexes are
usually instances, but the applications he describes
are to collections of document titles. The Encyclopedia
of Linguistics, Information and Control (2) mentions
only citation indexing.

This chapter is also exclusively concerned with back-of-the
book indexes; hereafter the term index will be used
in this restricted way.

The principal result presented here is an algorithm
for the automatic construction of an index from runr   g
text in machine readable form. A preliminary versi
of the algorithm was implemented by hand and used t
derive the index to Dolby, Forsyth, and Resnikoff (3).
The version presented here has been programmed for the
IBM 360/30 using a set of assembly code macros and
tested on a set of 50 abstracts of statistical papers
published in the Annals of Mathematical Statistics
and a second set of abstracts published in Cancer Research.

The difficult question of determining what is to
constitute an adequate index for a given corpus of
running text is not considered here, although reference
is made to an earlier study (Dolby (4)) that considered
certain obvious statistical characteristics of
published indexes as well as to the previous chapter.

The cost of deriving the index entries and formatting
them into standard format is approximately 2¢ per line
of input text, based on standard commerical rates (west
coast of the United States).

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Let us assume that an index is an ordered collection
of word sequences (or transformations thereof) from
the running text together with appropriate locator
designations (e.g. page numbers). A reasonable first
step in deriving such an index is to partition the
text into a set of word sequences using, in this case,
marks of punctuation and structure words to determine
the sequence boundaries.

Each sequence is then examined to determine whether
it should be deleted from the set. In particular,
sequences consisting of structure words only are deleted.
For reasons that will become evident later, sequences
consisting of single words and sequences that occur
only once in the entire corpus are also deleted from
the set.

Of the various possible transformations it is obvi-
ously desirable to identify singular and plural forms,
to invert certain word sequences (at least selectively)
so as to provide access to words occurring only at the
end of the word sequences, and to superimpose a "see"
and "see-also" facility to permit more complex transformation.

Implementation of such an algorithm requires repeated
access to various lists of words and morphemes. Computer
time will obviously be strongly influenced by the
strategies employed to accomplish these comparisons.
To cite the most obvious example, it is clearly more
efficient to store the list of structure words (which
is relatively small but contains many words of high
occurrence frequency) rather than the list of content
words which has the converse properties.

Where possible, significant gains can be made by
testing for word classes rather than for individual
words. Thus, it is useful to identify all participial
forms as these do not generally appear as index entries.
On the other hand, provision must be made to allow the
override of such rules for cases of particular importance.
(e.g. stratified sampling is an important statistical
entry that should not be suppressed.)

As the function of these various lists is primarily
to delete words from the index, it is convenient
to refer to the lists as "stop" lists and the sets of
override words as "go" lists. Although sufficient testing
on a wide variety of subject matter is not yet available,
it would appear that the stop lists are basically inde-
pendent of subject material and the go lists are subject

Figure 5.1

Word Frequency versus Rank

Brown University Standard Corpus of

American English

dependent. Thus a careful study of available authority
lists in the subject field would be necessary to insure
proper operation of the algorithm. (Such a study would
be necessary in any event to prepare the "see" and "see-
also" entries.)

Preliminary segment boundaries are established by
marks of punctuation (other than the hyphen and apostrophe).
Within the segments thus established, further boundaries
are introduced between sequences of consecutive stop
words (see Table 5.1) and non-stop words. As a simple
expedient, all words in the stop list ending in s
have the s removed and the match between the current word
and the stop list is made after the final s (if any) has
been removed from the current word. More sophisticated
"plural logic" would be justified here only if the
stop list were expanded substantially and in its
expanded form contained a significantly larger number
of "irregular" plurals.

The selection of the words to be used in the stop
list provides an intriguing problem. Clearly, all
structure words (neglecting archaic forms) should be
included. It turns out also to be useful to include
high frequency adjectives and verbs. It is therefore
tempting to simply select the first n words from a
rank ordered word frequency list. Unfortunately,
there is no clear break in such a list in the vicinity
of a reasonable cutoff (see Figure 5.1). Thus the
cutoff must be made simply in terms of finding a
reasonable trade off between added machine costs in
testing against large lists, and added editing costs
at the other end due to failure to suppress words.
Based on the developments of Chapter II, we would
expect the cutoff to be in the order of 1/30 of the
vocabulary. The word list used here has been purposely
kept short during programming and should probably be
expanded by a factor of two or three in actual use.

The list organization as presently implemented is
also quite simple: as the word length (in characters)
of the current word is known at the time of the match,
the list is broken down by word length and arranged
alphabetically, within the sets of each length. Matching
is done sequentially with termination on a match or
when the current word is low to the list. Expansion
of the lists would probably make it useful to use a
hashing technique.

The next segmentation stage consists of segmenting the
sequences of non-stop words into consecutive sequences
of words ending in ed, ly, ing, or ful and sequences of

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TABLE 5.1

SHORT LIST OF STOP WORDS
ARRANGED BY WORD LENGTH

an	own	some	three	general
at	put	such	under	improve
be	see	take	until	include
by	she	tend	usual	instead
do	the	term	where	operate
go	thi	than	which	present
ha	too	that	while	previou
he	two	them	whose	provide
hi	way	then	wider	require
if	who	upon	would	several
it	you	very	yield	similar
on		well		special
cr	also	were	become	through
me	back	what	before	unknown
my	been	when	behave	without
no	both	will	better	
so	ome	with	cannot	consider
to	uown	work	change	original
up	each	your	chosen	possible
wa	even		denote	satisfie
we	from	about	depend	together
	give	above	derive	
all	good	admit	discus	arbitrary
and	have	after	either	different
any	here	among	extend	excellent
are	hold	begin	higher	important
but	into	could	implie	otherwise
can	just	drawn	little	
did	know	first	permit	additional
due	last	found	relate	elementary
few	lead	given	reduce	particular
for	lend	great	result	
get	like	imply	second	
had	long	known	should	
her	made	might	unique	
him	make	never	variou	
how	many	other	wherea	
let	more	refer	within	
may	most	right		
new	much	sense	against	
not	must	shown	another	
now	only	since	because	
off	over	still	between	
old	part	their	certain	
one	said	there	consist	
our	same	these	earlier	
out	show	those	further	

N.B. All one-letter words are stopped. Terminal s is removed, thus ha stops has .

words not ending in any of those four suffixes. The
current go list to override this segmentation consists
of only three words (family, stratified, and sampling)
and is included only to insure that the facility
exists in the program.

The structure words of and in are not included in
the main stop list so as to allow sequences such as
analysis of variance and convergence in measure to
emerge as index sequences. However, it is clear that
primary index entries do not include entries beginning
or ending with of or in. Hence the final segmentation
step is to segment beginning or ending occurrences
of these to words from the non-stop, non-(ed, ly,
ing, ful) word sequences.

Following a suggestion of John Tukey, we have investigated
the utility of "stopping" all short words, i.e.,
words with fewer than a characters. Such a procedure
would clearly speed up the program and set aside the
difficulty of running down a number of short words
that occur with sufficient frequency so as to be included
in a reasonable system, (such as those occurring in
Latin phrases). Based on present experience, it appears
that suppressing words with fewer than four characters
is reasonable. This procedure has been used in the
experimental run on the 50 abstracts from Cancer Research,
but not on the two earlier examples presented here.

All segments other than those consisting wholly of non-
stop, non-(ed, ly, ing, ful), with beginning and ending
of and in removed, are deleted. Of the segments
remaining, all segments consisting of single words are
also deleted. Experimentation with this step in the
procedure stems from an observation made in Dolby (4)
that one word entries in published indexes occur with
surprisingly low frequency. Hence, the obvious
strategy is to suppress all entries with exceptions
rather than to pass all with exceptions.

The override to single-word suppression can take
several forms. First, a go list can be appended (though
none is used in the present implementation). Independence
would be an obvious choice for statistical subject
matter. Second, proper names, that is, words in all
caps or initial caps could be used as an override.
(This was done in the manually implemented version used
on Ref. (3) but has not been exercised in the machine
implementation.) Finally, single-word primary entries
can, and do occur in the inverted entries studied below.

This reduced list of segments, or possible index
entries, must now be transformed in certain obvious
ways both to achieve proper compression in the final
index and to provide at least the appearance of a
manually prepared index. One obvious consideration

involves the problem of identifying singular and plural
forms. Again, a relatively simple strategy is sufficient
to take care of most of the problem. Plural forms are
rarely used as modifiers and when so used are used with
a high degree of consistency. Thus if least squares
method occurs, it is highly unlikely that least square
method will also occur (though least squares methods
might well occur). Hence it is only necessary to
prepare for plurals that occur at the end of the entry.

The most frequently occurring plural form is obtained
by adding s to the singular form. If the final s is
replaced by a code that will sort immediately after
blank (but prior to a) it is possible to compare
successive entries after sorting and to eliminate the
final s from all entries that follow entries that
are otherwise identical. The final s is then restored
in all other cases. In the application to the statistical
abstracts 311 of the 946 entries ended in s. Of
these, 41 were stripped of the final s to provide
the required identification. More sophisticated logic
of the same variety could be added to handle plural
forms such as processes, densities, and matrices although
a quick survey of the 946 entries disclosed only four
such occurrences where identification was desirable.

Another purely manipulative step that must be introduced
at this stage is the generation of inverted entries
to provide access to words occurring at the end of the
text ordered entries. There appear to be two main
forms of interest. The first, typified by analysis
of variance, can be implemented by the obvious algorithm
that produces variance, analysis of. A more sophisticated
form could be used to suppress one or the other of the
two variants. A pair of relatively short, subject
dependent, stop lists would probably suffice for this
purpose.

A second type of inversion, typified by mapping normal
distribution into distribution, normal could either be
implemented by a go list of modest proportions or by
ordering the entire set of entries by last word and
then inverting all sets involving a common last word
of sufficiently high frequency. Neither of these alterna-
tives have been tried at this time, though some statistics
have been gathered on the behavior of statistical terms
from this point of view.

In addition to the deletion of one-word entries, it
is evident, when one operates on full text, that it is
entirely safe, and indeed quite useful, to delete
entries that occur only once in the text. Intuitively,
one can argue that if a term is not mentioned at
least twice (allowing for plural variants and the like)

then there is little likelihood that enough information
is presented about that entry to make it worthwhile
as an entry in the final index. Practically, an
examination of singly occurring entries in the samples
we have studied thus far makes it clear that this is
a highly useful device for eliminating much of the
"noise" that inevitably is present when one takes such
a simple view of English syntax. Statistically, the
step can be justified on the grounds that the resultant
index is of the proper size (as a percent of the volume
of the book indexed) when such entries are left out,
but noticeably too large if they are left in.

The use of this device must be tempered by knowledge
of the text. For instance, this device was not used
in the index to the statistical abstracts, as it was
evident that the abstracts did not possess sufficient
redundancy to allow proper operation of such a mechanism
within an abstract, and it seemed unwise to base the
use of such a mechanism on a (not necessarily homogeneous)
set of abstracts. Presumably there are certain books whose
text has a very low redundancy; for these this type of
deletum should not be impletmented.

The manual implementation of the algorithm on book
length material (reference (3)) is shown in Figure 5.2.
Two systematic departures from the general algorithm
were made in implementing it: first, names of States
were systematically deleted from the index; second,
a list of special words for inclusion in the index was
used, containing names of countries and languages.
Both decisions insure uniformity of in- or ex- clusion
of terms in each class without regard to the relative
significance of each usage. Finally, as described
in the Instructions for Use of the Index, two index
terms were manually inserted: the collective Computer
Languages, and the alternative World War I for the
algorithmically occurring First World War.

Perhaps of greater theoretical interest than those terms
that appear in the index in Figure 5.2 are those terms
that were deleted by the requirement that each entry that
appears in the index, except for entries having special
format properties, refer to more than one location in
the text. Table 2.4 lists those word sequences which
were excluded from the index for this reason. Preceding
some of the words are letters which describe properties
of the word sequence: 'p' indicates that the sequence
is a plural form of another word sequence selected by
previous steps of the algorithm; the plural sequence
is therefore equivalent to the singular one, and hence
appears in the final index. Sequences preceded by 'i'
appeared in italic type font. It appears that this font

Index

Instructions for Use of the Index

The index is the result of applying an algorithm to the text of the book; a minimal amount of (probably mechanizable) subjective human post-editing in the final two steps produced the amalgamated and reordered form that is printed below.

All word sequences that are not printed in italics appear in the given form in the text of the book, apart from possible differences of capitalization. Terms that do not explicitly appear in the text do not appear as index terms with the exception of the collective Computer Languages, and the alternative World War I for the naturally occurring entry "First World War."

Those readers who are experts in information retrieval and automatic indexing may be interested to know that this is a 4 percent index.

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Figure 5.2

ALGORITHMIC INDEX TO REFERENCE (3)

TABLE 5.2

Excluded Index Terms Referring to One Location
======== ===== ===== ========= == === ========

abnormal parenthesization
absolute frequencies
academic staff
access capability
access point
access system
accessible estimates
p accession distributions
accessions growth rate
acquisition rates
acquisitions data
acquisition mechanisms
acquisition process
acquisition rate
acquisition schedule
acquisition shares
acquisition structure
acquisitions - GNP relation
acquisitions - GNP share equality
acquisitions budget
p acquisitions growth
p acquisitions expenditures
adequate user access
algebraic equations
algebraic expressions
alpha-numeric code
alphabetic code
approximate linearity
approximate normality
p archival collections
archival holdings
archival libraries
archival records
        Assembly code programming
"assembly languages"
assignment procedure
author access
author field
author list
author name
author/title list
p authority lists
automated catalog
Auxiliary memory
average cost
average growth
average number
average record length
average time

p Baltimore County Libraries
bedroom states
biases inherent
p bibliographic descriptions
bibliographic holdings
bibliographic indications
bibliographic items
bibliographic listings
bibliographic lists
bibliographic notes
bibliographic practice
p bibliographic records
bibliographic references
bibliographical information
bibliographically incomplete
bibliography
Bibliography Field
bibliography section
book-publication depressions
Book Length
bookseller
budget dollar
budgetary requirements

business community
calculus text
call number

(Canadian)census figures
capital letters
capitalization conventions
capitalization errors
capitalization requirements
card catalog collection
card collection
card files
card space convention
card system
cards per entry
cards per title
"careful" study
case alphabet
catalog card conversion errors
p catalog cards
catalog data
p catalog files
catalog interrogations
catalog preparation
catalog productions

error-correction capability
European
executable statements
expansion ratio
"explosive" growth
exponential curve
exponential expansion
exponential function
exponential imprint date distribution
exponential library growth rates
exponential rate
faculty library committee
feedback response
field names
i fields
fields per record
file figures
file maintenance
file records
file structure
file system
financial data
financial community
financial transactions
(first) generation machines
fixed absolute growth
floating point arithmetic
follow-up correspondence
foreign language acquisitions
foreign language documents
foreign titles
Format-Dependent Errors
format capabilities
format compromise
Format control
format elements
format requirements
French-African
French-speaking
functional collection
fund name
fundamental processes affecting libraries
fundamental structure
future funding needs
geographic area
geometric decrease
global category
global check
global war
GNP-acquisitions relation

GNP at Market Prices
graph paper
graphic arts
graphic representation
Gross Domestic Product
p gross national products
Gross Personal Income
ground-level extension
growth challenge
growth periods
growth phenomenon
growth problems
p growth rates
growth statistics
hardware costs
p Harvard samples
"higher level" languages
historical events
historical significance
human costs
human readable document
identification number
illegitimate code
implementation cost
imprint data
p imprint dates
imprint date growth
imprint decade
p imprint distributions
in-depth studies
in-school access
income data
income growth
income ratios
indented lines
information base
information fields
information per inch
information per page
information run-over
input costs
input errors
input format
input methods
input program
inquiries per record
instruction per second
intercolumn space
interentry blank lines
interlibrary loan service

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machine use
magnetic cores
magnetic discs
magnetic drums
p magnetic tapes
main file
management tool
manipulative operations
manpower costs
manual generation
manual operations
manual strategies
manuscript form
map classification category
marginal improvements
mathematical computation
mathematical exercise
Mathematical Journal Titles
mathematics
mathematics faculty committee
mathematics journals
mean growth
mechanical errors
mechanical translation
mechanization context
methodological principle
i Misspelled words
model cost equations
monetary inflation
monograph collection
monographic letter frequencies
i month portion
multi-language manipulation procedures
multicharacter vowel string
multiple copy graphic arts quality author list
musical scores
national accounts statistics
national economic growth
national economy
national origins
national publications
national publishers
national statistcial data collection processes
natural languages
(new) acquisitions information
non-English words
non-numerical procedures
nonlibrary customers
nonlinear scales

nonoriental monograph acquisitions
nonpamphlet items
nonserial Fondren sample
nonserial shelf list cards
nonserial textual works
nonstationary growth periods
nonstationary intervals of
    library growth
non stationary time series
"normal probability paper"
normal distribution
normal probability distribution
normative measures
number field
numeric symbols
numerical computation
off-site areas
on-line input
open-stack libraries
optical character recognition
    equipment
optional parameters
order-of-magnitude changes
order date
order file records
(order of) magnitude cost reductions
(order of) magnitude cost variations
(order of) magnitude decisions
(order of) magnitude gains
order operation
order system
order system file
order system reports
ordinal numbers
out-of-date catalog
output error signals
output list
output machines
output printers
output sheet
page counts
page design
paper costs
(paper) tape input
parallel search logic
pattern-matching facilities
pattern-valued functions
pattern primitives
per capita growth

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per unit basis
percentage growth
personal author
personal authorship
p personal incomes
photo-offset reproduction
(physical) volumes per serial title
pilot study
plant expansion
political disintegration
political issues
population growth
potential control
print runs
printed copies
(printing and) binding costs
printing cost
i Printing Type Faces
private endowment funds
Private Finance
probability scale graph
process flow
x processing bibliographic records
x processing linguistic information
production costs
production economies
production processes
productivity per dollar
profound machine language level study
program errors
program routines
proper-name entries
i Proper names
proper scale compression
propositional calculus"
public acceptance
public card catalog
public catalog losses
p public libraries
public sales
public use
publication cost
i publication field
publication growth
publishing industries
punch paper tape
quality performance figures
quality point
quantal jump characteristics
quantal jumps
i random access

p random samples
record entry
recursive processes
refugee movements
relative frequencies
relative frequecy distribution
relative merit
relative performance
relative significance
relative size
reliable data
rental figure
report system
x reprogramming costs
research effort
research grants
research purposes
retrieval processes
retrieval requests
p retrospective files
p retrospective materials
run costs
salary structure
sample cards
school cooperation
scientific effort
scientific machines
scientific periodical literature
scientific publication
scientific research
selection criteria
selection operation
selection procedure
selection processes
selection technique
semibold type faces
semilogarithmic paper
p serial publications
serials shelf list
service bureaus
set theoretic operations
share distribution
shelf list circulation file
Shelf List Statistics
shelf space
significant acquisitions - GNP
disagreement
social dislocation
social ideologies
social phenomena
social systems

does not characterize indexible sequences. 'x' indicates
that a manual error has been made; in some cases a verb
gerund has not been deleted in the stop list step of
the algorithm, so a sequence appears in the later stages
of the algorithm when it ought to have been deleted at the
first stage. For example, the sequence processing
bibliographic records contains the structural stop
sequence -ing indicating the gerund form; exclusion of this
word at the stop list stage would have left the subsequence
bibliographic records for consideration, which appears in the
index anyway because it occurred in more than one additional
location. The indicator 'e' means that the sequence has been
excluded by the human posteditor. Two such sequences are
noted: square inch, which should perhaps inhabit the
stop list, and XYZ Library, which must be considered because
one of the special format inclusion conditions is that
sequences containing all capitalized words are indexed
regardless of the number of text locations to which they
refer; but this instance doesn't supply any useful information.
It is a stylistic curiosity. Finally, certain sequences
in the table are preceded by a parenthesized word. For
instance, (first) generation machines appears. The algo-
rithm generated generation machines; the preceding text
word was included in the list to help the reader to under-
stand the context of the sequence, which, following the
algorithm, was excluded from the index.

Quantitatively, this algorithmic index is not significantly
different from the manually produced indexes analyzed in
Chapter IV. The gross size of the index is 5 pages as
compared to 157 pages of text, a text to index ratio of
31.4 to 1. The index entry length distribution is given
in Table 5.3.

Table 5.3

Index Entry Length Distribution
Computerized Library Catalogs

Number of Words	Frequency	Cumulative Frequency
1	82	82
2	190	272
3	44	316
4	13	329
5	6	335
6	4	339
7	1	340

The percentage of one-word entries (24%) is higher than the average number of one-word entries in the subsample from the Fondren Index Sample (13%). Although this is not a significant deviation (more than 17% of the indexes in the subsample had more than 24% one-word entries) it is worthy of some comment: the basic algorithm suppresses one-word entries, with exceptions. In this case the exception rule was to include capitalized one word entries. Thus, even though the algorithm is designed to operate against one-word entries, the proportion occurring is still on the high side.

The distribution of entries by number of words is shown in Figure 5.3. The distribution is reasonably approximated by the lognormal distribution. The arithmetic mean of the distribution is 2.08 words per entry, compared to 3.68 words per entry for the subsample as a whole. Although there is again some cause to question whether this is a significant deviation, there is an underlying weakness in the form of the algorithm as it was used in this example. The algorithm excludes entries of the word X of Y. In (4) the structure-word-free entries were found to have a mean number of words per entry of 2.12, almost exactly the average found for this algorithmic index. However, the structure-word-free entries of (4) made up only 55% of the total number of entries. In Chapter VI we shall return to this question in analyzing the output of the basic algorithm where the capability to generate entries of the form X of Y has been included.

The absence of structure-word entries also tends to depress the overall size of the index. Although the bulk size, measured in pages is approximately 1/30th of the text size (as would be expected), the ratio of bulk of the index to bulk of the text measured in number of characters is approximately half of this figure. (Not only are the index entries somewhat shorter than would be found in the manual indexes, the text density is approximately 3,150 characters per page as compared to the mean of 2,400 characters per page.)

The lack of structure-word entries also tends to distort the page location distribution, (Table 5.4).

Figure 5.3

Index Entry Length Distribution
from the Algorithmic Index
to Computerized Library Catalogs

Number of Words per Entry

PERCENTAGE

PROBITS

Table 5.4

INDEX PAGE LOCATION DISTRIBUTION
COMPUTERIZED LIBRARY CATALOGS

Number of Page Locations Per Entry	Frequency	Cumulative Frequency
1	127	127
2	102	229
3	52	281
4	18	299
5	11	310
6	8	318
7	6	324
8	2	326
9	5	331
10	1	332
11	1	333
12	1	334
13	1	335
14	3	338
15	0	338
16	2	340

The graph of the index page location distribution is shown
in Figure 5.4. Here it is evident that the number of entry
with but a single page location is significantly lower than
the overall trend line for the rest of the data. Further,
the bend in the data occasioned by this low value is sharper
than for any of the distributions in the subsample from the
Fondren Index Sample (see Appendix II). Interconnection of
the entries with structure words would clearly tend to break
apart entries presently agglomerated, thus reducing the number
of multiply occurring entries. Ignoring the low number of
singly occurring entries, the Zipf-Mandelbrot slope is 2.17,
well within the range of values found for the manually
produced indexes.

The arithmetic mean of the number of page locations per
entry is 3.19, nearly double the figure found for the sub-
sample of the Fondren Index Sample. However, this value is
distorted by the fact that consecutive page locations were
not agglomerated into single locations as is normally done
in manual indexing. When this factor is corrected, the
average number of page locations per entry becomes 2.14.
As this value would be further reduced by inclusion of
structure-word entries, it would appear that this variation
is not at all significant.

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Figure 5.4

Index Page Location Distribution
from the Index to
<u>Computerized Library Catalogs</u>

NUMBER OF PAGE REFERENCES

In sum, aside from the failure to include structure-word
entries or to agglomerate consecutive page locations, the
statistical shape of the algorithmic index to Computerized
Library Catalogs appears sound. This is not to say that the
index is entirely comparable to a manually produced index.
However, the first requirement in automating a process
traditionally done manually is to meet the basic size
constraints. Further developments in the technique will
be illustrated in the next chapter to demonstrate that
even closer approximations are possible.

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CHAPTER VI

AMALGAMATIVE ACCESS MECHANISMS

AMALGAMATIVE ACCESS MECHANISMS

INTRODUCTION

The model proposed in Chapter 2 shows that the search
for access mechanisms must be conducted in compressive
powers of 30. It is principally the <u>relative size</u>
of an access mechanism that determines its utility.
That a compression of 30 must be effected in order to
move from one access level to the next, and that the
boundary between access levels corresponds to compression
of about a factor of 5 implies that there cannot be very
many possible access mechanisms to a particular level
of information storage. For instance, if the level to
be accessed is the book, then one must ask what natural
subsets of information there are in a book which consti-
tute about one-thirtieth of it. As has already been
pointed out, the average index to the average book
compresses the text by a factor of 31.8, so the book
index is a viable access mechanism. Studies of abstracts
of papers appearing in mathematical journals show that
the average complete abstract produces a compression of
about 30.6, so the journal paper abstract is also a
viable access mechanism. The <u>book abstract</u> should require
about $276.6/(2e)^2 = 9.3$ pages; we do not have reliable
information about the average length of book reviews in
the professional literature, but this appears to us to
be a possible mean for scholarly reviews. On the other
hand, the capsule reviews of popular books that appear
in newspapers and other popular media, and in some schol-
arly publications, are much shorter--perhaps the equiva-
lent of one or two pages--and lie on the boundary between
the levels of access mechanisms to books and access
mechanisms to access mechanisms to books, the latter
operating at the level of an enlarged table of contents
such as regularly appeared in previous centuries, and
still sometimes do, <u>viz</u>., Hans Zinsser's <u>Rats, Lice and
History</u>'s table of contents from which we extract the
following:

I.   In the nature of an explanation and an
     apology

II. Being a discussion of the relationship
     between science and art

III. Leading up to the definition of bacteria
     and other parasites, and digressing briefly
     into the question of the origin of life

IV. On parasitism in general, and on the neces-
     sity of considering the changing nature of
     infectious diseases in the historical study
     of peidemics

V.   Being a continuation of Chapter IV, but
     dealing more particularly with so-called new
     diseases and with some that have disappeared.

and so forth.

Another way of looking at the problem of discovering
possible methods for accessing books is this: the number
of characters in a book is about $(2e)^8$; reduction of a
factor of $(2e)^2$ leads to an information store about the
size of the index; further reduction by a factor of $(2e)^2$
to the next access level leads to a store of the size
of the table of contents. Another reduction by the same
factor produces $(2e)^2 \cong 30$ characters, which is nearly
the size of a book title, as we have determined in a
preliminary fashion from a small uniform subsample of
the Fondren Sample. In fact, that estimate was 34.2
characters for monographs in the sample regardless of
language of title; had the subsample been restricted to
English language titles, the average length would have
been shorter. A final usable reduction is effected by
another division by $(2e)^2$, leading to ، one character
access mechanism such as that provided by the Library
of Congress one letter class designation.

The important point is that every access level is filled.
Further study of possible new access mechanisms must
therefore be constrained to access mechanisms of the same
size as those that already exist. A natural question that
arises is whether it is desirable to have two access
mechanisms of the same size for a particular information
system. That such duplication does already exist is
easy to demonstrate:

1. The Author, Title, and Shelf orderings or a library card catalog are all essentially of the same size: roughly, one card image for each title in the collection. (The subject heading ordering is generally slightly larger, but still at the same access level as the others.)

2. The table of contents for a book is at the same access level as the catalog record.

3. Abstracts to journal articles appear in abstract journals as well as the index entries that are frequently published at the end of the year in the journal. Both of these access mechanisms are first order devices.

and of course other examples involving titles, descriptors, etc. can easily be found.

Thus the size of an access mechanism, though it is of first importance in describing the nature of the access it provides, is not sufficient to completely describe its characteristics. A second consideration that must be taken into account is easily illustrated by considering the sequences:

Article, Abstract, Title

and

Book, Index, Table of Contents

In the first sequence, each access device is acting simply to compress the contents of the primary information store. In the second sequence, each access mechanism is itself a set of lower order access mechanisms collected and sorted in a useful ordering. The abstract and the title provide the user with the opportunity to determine whether the document so described is likely to be relevant to his need for information, in a general way. The index and the table of contents provide the user with information about the contents of the document together with the location of particular pieces of information in the document.

The crucial question is that of agglomeration: an index is an agglomeration of entries; a table of contents is an agglomeration of entries; on the other hand both title and the abstract are entities themselves rather than being agglomerations of other entities. It seems clear from what has gone before that the minimal unit for

agglomeration is the first level unit (about 30 characters). Thus both the table of contents and the index are agglomerations of first level units. However, higher level agglomerations exist: the abstract journal is an agglomeration of second level units, as is a publication devoted to the republication of the tables of contents of journals. Although we have not yet completed our study of dictionaries and encylopedias, it is clear that each of these important access devices are agglomerations of higher level entities.

In this sense, an access mechanism can be described first by its total size and secondly by the size of the primary entries that it agglomerates. Thus an abstract is zero level agglomeration of second level entries; a table of contents is a first level agglomeration of first level entries; and an index (to a book) is a second level agglomeration of first level entries.

There are at least two other factors that must be taken into account: a cumulative index to a series of books on statistics obviously plays a different role than the index to an encyclopedia even though both are third level agglomerations of first level entries. The difference here is that the encyclopedia is itself an agglomeration of second or higher level access mechanisms, while the books are primary information stores. The difference in these two mechanisms would almost undoubtedly show up in the slope (in the Mandelbrot sense discussed earlier) of the index.

Finally, there are access mechanisms clearly dedicated to "non-subject" access, e.g., author indexes, list of publications by publisher, place of publication, time of publication, etc. which play a major role in library access systems.

Consider a collection of titles--such as book titles--of items which corpass a range of subject matter. The card catalog title list is one ordering of such a collection. If the collection is reordered to bring together all titles which contain a given information bearing word, then access to the collection is significantly increased.

Studies of such access mechanisms have been underway for some time, although none of them are generally available. One of the most advanced title access mechanisms is that prepared at Princeton University under the direction of J. W. Tukey; it is a sophisticated permuted title index consisting of more than 25,000 titles of journal papers in the field of statistics. Since the average length of a paper in mathematics is about 13.8 (normalized) pages,

a title represents a compression of about two access
levels, for the title as it appears in a permuted title
index carries information about the journal and author
as well, requiring about 130 characters. A sample page
from the Princeton permuted title index is shown in
Figure 6.1.

General considerations suggest that a permuted title list
of book titles for the Library of Congress letter class
subcollections of rchival libraries would be a useful
tool, and one whi. would be readily obtainable as a
byproduct of the existence of a machinable catalog
data base.

Another type of amalgamative access mechanism, which
provides access to a collection of items belonging to
the same access level rather than to only one item can
be constructed by performing the process normally used
to construct a standard access mechanism on the output
produced by another. For instance, we have studied the
utility of indexing abstracts to journal papers in the
statistical literature. The abstracts are normally
provided with the papers; they have been converted to
machinable form and an elementary version of the indexing
algorithm described in Chapter 5 was applied to them.
Appendix A5 exhibits the abstracts to 50 papers, the
associated abstract indexes produced by application of the
algorithm, and a cumulative list of the resulting index
terms with references to the articles in which the terms
appeared. We reproduce an abstract with its index as
Figure 6.2 and a page from the cumulative abstract index
as Figure 6.3. The abstract index was the first processed
in this series; it is perhaps not entirely typical of
the output from the algorithm. We have also processed
the same data using a variant of the algorithm which
ignores in its analysis stage the presence of the preposi-
tion "of" and consequently will produce index entries
like "basic limit theorem of renewal theory" which appears
in Figure 6.2 only by way of its constituent phrases
"basic limit theorem" and "renewal theory".

An index to an abstract is a hybrid form of access
mechanism. The abstract already contains a large propor-
tion of significant phrases which are repeated in the
extractive output of the indexing algorithm. There is
therefore no hope that an index to an abstract can provide
a compression of a full factor of 30 that would be
necessary to descend from one access level to that immedi-
ately below it. In fact it appears that indexing abstracts
leads to a compression of about 15; since this is
significantly greater than (2e), such a procedure does

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Figure 6.1
Permuted Title Index Page (Left Hand Side)

Figure 6.1
Permuted Title Index Page (Right Hand Side)

```
.. BUREAU OF THE CENSUS. /ICATIONS OF ELECTRONIC EQUIPME
5. DATTT.
5 LINEAR RESTRAINTS. / FUNCTION OF A WEIGHTED SUM OF NON-
5. AFRICT.                              THE EFFICIENCY OF
5. WILKS = STATESMAN OF STATISTICS.
5 FOR A FAIR OF TIED RANKINGS.
AANGESTED INGEN.                                        DE SP
AANHEMELIJKE VERDELINGEN.
AANPASSEN VAN FUNCTIES AAN EEN GROOT AANTAL WAARNEMINGSUI
AANPASSING VAN POLYNOME AAN IN REEKS EENTIJDIGE GEGEVENS.
AANSLUITING OP EEN VARIANTIE-ANALYSE.
AANTAL INCONSISTENTIES IN EEN BEPAALD RANGCORRELATIESCHE/
AANTAL KANSEN.
AANTAL KANSEN.                              VERGELIJKING VAN
AANTAL NOODZAKELIJKE WAARNEMINGEN BIJ TOEPASSING VAN VOOR
AANTAL STEEKPROEF GREEDTES. / SPREIDING VAN EEN NORMALE V
AANTAL WAARNEMINGSUITKOMSTEN.                EEN OPMERKING O
AANTEKENINGEN BIJ DE HERZIENING VAN DE METHODIER DER PR
AANVOER EN OPSLAG VAN DUNNE MELK IN EEN MELKPRODUCTENFA
AANWIJZEN VAN UITBIJTERS.
A+B IN TERMS OF ONE WITH COVARIANCE MATRIX A. /ING IN
ABAC FOR TESTING THE SIGNIFICANCE OF RHO.
ABAC FOR THE SAMPLE RANGE.
ABACS FOR THE RAPID ESTIMATION OF A TETRACHORIC COEFFIC
ABACS FOR DETERMINATION OF A CORRELATION COEFFICIENT CO
ABAC FOR DETERMINING THE MEAN DEVIATION OF A CLASS FRO
ABAC FOR DETERMINING THE PROBABLE ERRORS OF CORRELATION C
ABACS FOR ITEM-TEST CORRELATION AND CRITICAL RATIO OF
ABANDONING AN EXPERIMENT PRIOR TO COMPLETION.
ABBREVIATED EDGEWORTH AND GRAM-CHARLIER SERIES.     THE
ABBREVIATED PROCEDURE FOR DERIVING EXPECTATIONS OF SUMS O
A.B.C. OLNE A.B-VERSUCH.
A.B.C.-FAKTORENVERSUCH MIT UNGLEICHEN KLASSENFREQUENZEN.
ABELIAN GROUPS.
ABELIAN GROUP CHARACTERS.                     ON THE CONSTRUCTIO
ABELIEN. /ACTORIELS FRACTIONNES ET CERTAINS CODES CORRECT
ABERRANT COMPARISONS )
ABGANGENIE FÜR WIRTSCHAFTLICHE UND TECHNISCHE GESAMTHEIT
ABGANGSORDNUNG VON AKTIENGESELLSCHAFTEN.
ABGRENZUNG EINSEITIGER TOLERANZBEREICHE MIT HILFE VON MIT
ABHANGIGEN BEOBACHTUNGEN.
ABHANGIGKEIT DER WEITSPRUNGLEISTUNGEN VON ALTER UND KORP/
ABHANGIGKEIT VON DER ANZAHL DER INITIAL ZELLEN DES S/
ABILITA DEI GIOCATORI NELLE CORSE AL GALOPPO.
ABILITIES OF SOME TEN-YEAR-OLD TWINS.
ABILITIES.
ABILITIES.
ABILITIES.
ABILITIES.
ABILITIES AND PERSONALITY TRAITS.
ABILITIES.
ABILITIES TO ACTIVITY PREFERENCES.
ABILITIES TEST BATTERY. /REVISED ORTHOGONAL ROTATIONAL SOL
ABILITIES TESTS.                              A FACTORIAL
ABILITIES STUDY.     APPLICATION OF THE QUARTIMAX MET
ABILITIES.                         THE APPLICATION OF MUL
ABILITIES.
```

Figure 6.2

Abstract and Abstract Index

WILLIAM FELLER
AN INTRODUCTION TO PROBABILITY THEORY AND ITS APPLICATIONS, VOLUME I, 3RD ED.
CHARLES J. STONE, UCLA

THERE ARE A NUMBER OF SIGNIFICANT CHANGES IN FELLER'S EXCELLENT AND UNIQUE
INTRODUCTION TO PROBABILITY THEORY. THE CHARACTER OF THE BOOK, HOWEVER, REMAINS
FAITHFUL TO THE ORIGINAL EDITION.
ACCORDING TO FELLER IN HIS PREFACE TO THIS EDITION, THE GREATEST CHANGE IS
IN THE CHAPTER ON FLUCTUATION THEORY. THIS CHAPTER WAS INTRODUCED ONLY IN THE
SECOND EDITION, WHICH WAS IN FACT MOTIVATED PRINCIPALLY BY THE UNEXPECTED
DISCOVERY THAT ITS ENTICING MATERIAL COULD BE TREATED BY ELEMENTARY METHODS.
BUT THIS TREATMENT STILL DEPENDED ON COMBINATORIAL ARTIFICES WHICH HAVE NOW
BEEN REPLACED BY SIMPLER AND MORE NATURAL PROBABILISTIC ARGUMENTS. IN ESSENCE
THIS CHAPTER IS NEW.
OTHER CHANGES POINTED OUT BY FELLER IN HIS PREFACE OR NOTED BY THE REVIEWER
INCLUDE A BETTER TREATMENT OF INDEPENDENT TRIALS, RESTATEMENTS OF SOME OF THE
RESULTS ON THE NORMAL APPROXIMATION TO THE BINOMIAL DISTRIBUTION. SOME
ADDITIONAL MATERIAL ON BRANCHING PROCESSES. A MORE DETAILED PROOF OF THE BASIC
LIMIT THEOREM OF RENEWAL THEORY, AND A NEW SECTION IN THE CHAPTER ON MARKOV
CHAINS COVERING TABOO PROBABILITIES AND THE RATIO LIMIT THEOREM FOR NULL
RECURRENT CHAINS.

NUMBER OF SIGNIFICANT
PROBABILITY THEORY
FLUCTUATION THEORY
COMBINATORIAL ARTIFICES
NATURAL PROBABILISTIC ARGUMENTS
TREATMENT OF INDEPENDENT TRIALS
NORMAL APPROXIMATION
BINOMIAL DISTRIBUTION
BASIC LIMIT THEOREM OF RENEWAL THEORY
MARKOV CHAINS
TABOO PROBABILITIES
RATIO LIMIT THEOREM
NULL RECURRENT CHAINS

Figure 6.3

Cumulative Index to 50 Abstracts (one page)

realize a compressive gain that may be useful for accessing the abstracts. It will certainly be useful for accessing the original documents when it is applied to a collection of abstracts and the resulting indexes are accumulated.

The page extracted from the middle of the cumulative abstract indexed reproduced as Figure 6.3 shows that one paper in the sample of 50 referred, via its abstract, to the "NON-CENTRAL MULTIVARIATE BETA DISTRIBUTION", and, since the abstract transmitted this phrase, the paper undoubtedly contains something of interest about this topic. Similarly note that eight papers referred to the "NORMAL" distribution in some form. The presence of spurious terms like "ONTO ITSELF" and "OPTIMUM BLUE'S" is no more than a minor annoyance in use of the index, and is of course due to inadequacies in the indexing algorithm's "stop list", which should certainly contain the word "ITSELF". There are other more subtle problems whose genesis is the indexing algorithm, but they are not so obtrusive as to mae the use of the list burdensome. For instance, the phrase "OPTIMUM BLUE'S occurs in the abstract, where it is defined to denote "OPTIMUM BEST LINEAR UNBIASED ESTIMATE"; this phrase certainly belongs in the index, but it is not clear that a user of the amalgammated index would recognize the technical meaning of "BLUE" until it had become a standard term of the field.

Indexing abstracts is of potential value in gaining access to the large numbers of journal papers which annually appear in the literature; coupled with permuted title access mechanisms, the abstract index should provide a rapid and reliable means of surveying the key content areas of papers without the time-consuming process of reading abstracts, which often limits one to a relatively narrow and current range of documents.

When compared to the earlier manual implementation of the algorithm on Computerized Library Catalogs, the machine implementation of the algorithm differs in several ways, aside from the obvious fact that the machine is entirely consistent in its application where manual procedures cannot be. The raw data for the machine test on the statistical abstracts was keypunched in all upper case, as a matter of convenience. Hence, the rule to keep capitalized one-word entries was inoperative in this run. Further, no attempt has been made to include see or see-also types references in the machine implementation. On the other hand, the machine implementation includes logic to allow structure-word entries where the manual implementation did not.

These differences are reflected in the statistics describing the entry length and page location distributions.

Table 6.1 provides the entry length (in number of words) distribution for the machine index to the statistical abstracts.

Table 6.1

Entry Length Distribution
Algorithmic Index to Statistical
Abstracts

Number of Words	Frequency	Cumulative Frequency
1	0	0
2	315	315
3	233	548
4	125	673
5	54	727
6	19	746
7	6	752
8	2	754

Comparing this distribution to the comparable distribution for the manually implemented algorithmic index to Computerized Library Catalogs (Table 5.3) one sees that the proportion of one-word-entries has been reduced to zero (because there is no logic available to permit one-word-entries) and that the overall average entry length has been increased from 2.08 words per entry to 3.01 words per entry. The main factor in this increase is the introduction of structure-word entries, although the absence of one-word-entries has a small effect on average entry length as well.

The entry length distribution is plotted on Figure 6.4. Despite the absence of one-word-entries, the points are nicely fit by a straight line confirming the nice approximation by a lognormal distribution.

It will be recalled that in the previous study of page location distribution for the manually implemented version of the algorithm on Computerized Library Catalogs there was a significant bend in the Zipf-Mandelbrot straight line due to either a reduced number of singly occurring entries, or an excessive number of multiply occurring entries. For the machine version of the algorithm the page location distribution (or, more accurately, the abstract number location distribution) does not show this deviation (see Figure 6.5). The distribution is given in Table 6.2.

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Figure 6.4

Entry Length Distribution
Algorithmic Index to
Statistical Abstracts

PERCENTAGE

PROBITS

Number of Words per Entry

KEUFFEL & ESSER CO.

Figure 6.5

Abstract Number Location Distribution
Algorithmic Index to Statistical Abstracts

Number of Abstract References

Table 6.2

Abstract Number Location Distri-
bution, Algorithmic Index   to
Statistical Abstracts

Number of Abstract Locations per Entry	Frequency	Cumulative Frequency
1	703	703
2	31	734
3	6	740
4	9	749
5	5	7ь4

When compared to the comparable data for the manual
implementation, it is clear that not only has the
difficulty of an insufficient proportion of singly
occurring entries been been corrected by the inser-
tion cf structure word logic, but the slope of the
line has been significantly increased from 2.17
to 4.49.   This increased slope can of course be
attributed in part to the nature of the material
covered in the two cases and, perhaps, in greater
proportion to the structure of the material (i.e.
fifty abstracts vs. a single text).   Nonetheless,
the increase in slope does tend to confirm the ex-
pectation that use of structure-word entries is de-
sirable to increase slope.

Potentially more useful than the amalgamation of indexes
to abstracts to papers or books is the amalgamation of
indexes to the primary texts themselves.   We have under-
taken an extensive project designed to provide a realistic
test of the utility of amalgamations of book indexes
as well as an indication of the problems that would be
encountered in the preparation of such access mechanisms.

The indexes contained in 80 books on statistics have been
committed to machinable form.   Approximately 30,000
index entries (not all of which are distinct) are repre-
sented, which is nearly 400 entries per book.   This is sig-
nificantly less than the average of 838 index entries per
book obtained from the Fondren Index Sample, but, as is clear
from Table 4.4, it is well within the deviations typically

obtained by restriction of a sample to small and
specially defined subsets. We have not attempted to
determine the average number of pages per book in this
statistics sample; it may well be that the average number
of index entries per page is in closer agreement with
the figure obtained for the Fondren Index Sample.

The Statistics Index Sample is currently in the early
stages of amalgamation. In this report we can only
exhibit a combined alphabetically ordered list which
has not been formatted (to reproduce the usual format
of a book index) and which exhibits the consequences
of some program "bugs" not yet corrected which result
in the replication of input records at various places
throughout the amalgamated index. In spite of these
difficulties, the amalgamated list is already a valuable
access tool.

Table 6.3 lists the books that constitute the Statistical
Index Sample. The code in the leftmost column is the
abbreviation for the book used in the amalgamated index.
These books were chosen by a professional statistician
as representative of the more important information in the
statistics field that is available in monograph form.
The choice of 80 books rather than a larger number is
purely conventional; continuation of this project will
increase the data base and permit us to determine how the
yield of new index terms varies with increasing size
of the sample.

Following the lead of the analysis of the structure of
the index to a single book given in Chapters 3 and 4,
we see that the rank-frequency distribution Figure 6.6
is just another form of the index reference distribution
discussed in those chapters; in the form shown here,
the abstract entries appear at the top left part of the
graph, and the horizontal portions of the graph correspond
to those entries which refer to the same number of text
locations. Consequently, the abstract entries for the
Statistics Sample certainly include those that have
ranks less than 30, and may include several more but
not any with rank greater than 50.

Table 6.4 lists the 30 index terms that refer to the
greatest number of pages; personal names have been placed
in the right hand column; otherwise the order of appearance
in the amalgamated index list is the order shown in the
table.* This list is a useful pedagogical tool, providing

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* The frequencies given here are very tentative, as no
attempt has yet been made to agglomerate proper names
appearing in variant form.

TABLE 6.3

Bibliographical Description of the Statistics Index Sample

A   Elementary Decision Theory - Chernoff
B   Nonparametric Methods in Statistics - Fraser
C   Statistical Methods for Chemists - W. J. Youden
D   Analysis of Straight-line Data - Acton
E   Testing Statistical Hypotheses - E. L. Lehmann
F   Introduction to Mathematical Statistics - Paul G. Hoel
G   The Design and Analysis of Experiments - O. Kempthorne
H   An Introduction to Multivariate Statistical Analysis - T. W. Anderson
I   Statistics--An Introduction - D. A. S. Fraser
J   Linear Computations - Paul S. Dwyer
K   Modern Probability Theory and Its Applications - Parzen
L   Planning of Experiments - Feller
M   Theory of Games and Statistical Decisions - Blackwell and Girshick
N   An Introduction to Probability Theory and its Applications, Vol 1 - Feller
O   Elementary Statistics - Paul G. Hoel
P   The Elements of Probability - Cramer
Q   Statistical Decision Theory - Weiss
R   Introduction to Probability and Random Variables - Wadsworth and Bryan
S   Introduction to the Theory of Statistics - Mood and Graybill
T   Elements of Probability and Statistics - Wolf
U   An Introduction to Linear Statistical Models, Vol. 1 - Graybill
V   Elements of the Theory of Markov Processes and Their Applications - Bharucha-Reid
W   Geometrical Probability - Kendall and Moran
X   Fundamentals of Statistical Reasoning - Quenouille
Y   Characteristic Functions - Lukas
Z   An Introduction to Probability Theory and Its Applications, Vol. 2 - Feller
AB   Elements of Mathematical Statistics - Alexander
AC   Statistical Theory and Methodology in Science and Engineering - Brownlee
AD   Statistics and Experimental Design, Vol 1 - Johnson and Leone
AE   Mathematical Statistics - Wilkes
AF   Experimental Designs - Cochran and Cox
AI   A Course in Probability Theory - Kai Lai Chung
AJ   Essentials of Probability - Arthur Yaspan
AK   The Design of Experiments - Fisher
AL   Computational Handbook of Statistics - Bruning and Kintz
AM   Design and Analysis of Experiments - Quenouille
AN   Handbook of Statistical Tables - Owen
AO   The Elements of Probability - Berman
AP   Design and Analysis of Industrial Experiments - Davies
AQ   Statistical Theory - Lindgren
AR   Introduction to Statistics - Carlborg
AS   Probability and Statistics - Adler and Rossler
AT   Measuring Uncertainty--An Elementary Introduction to Bayesian Statistics
AU   A Brief Introduction to Probability Theory
AV   Statistical Design and Analysis of Experiments for Development Research - Villars
AW   Statistics in Research - Ostle
AX   Schaums Outline Series Theory and Problems of Probability - Lipschutz
AY   Elementary Mathematical Programming - Metzger
AZ   Statistical Inference for Markov Processes - Billingsley

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Continued -

BC   Introduction to Probability--A Programmed Unit in Modern Mathematics
BD   Statistical Analysis of Stationary Time Series - Grenader and Rosenblatt
BE   Statistical Methods in Experimentation--An Introduction - Lacey
BF   Stochastic Processes--Basic Theory and Its Application - Prabhu
BG   Probability and Frequency - Plummer
BH   Statistical Methods for Research Workers
BI   Probability, an Intermediate Text-Book - Bixley
BJ   Regression Analysis - Williams
BK   Statistical Processes and Reliability Engineering - Chorafass
BL   Introduction to Probability and Mathematical Statistics - Birnbaum
BM   Elementary Mathematical Statistics - Baten
BN   Introduction to Biostatistics - Bancroft
BO   Sampling Techniques - Cochran
BP   A History of the Mathematical Theory of Probability - Todhunter
BQ   Statistical Methods in Biology - Bailey
BR   Statistical Theory - The Relationship of Probability Credibility and Error
BS   An Introduction to Multivariate Statistical Analysis - Anderson
BT   Probability and Experimental Errors in Science - Parratt
BU   Contributions to Order Statistics - Sarhan and Greenberg
BV   Introduction to Statistical Method - Ehrenfeld and Littauer
BW   Theory of Probability - Jeffreys
BX   Statistical Adjustment of Data - Deming
BY   Statistical Analysis in Chemistry and the Chemical Industry - Bennett and
                                                                        Franklin
BZ   Probability Random Variables and Stochastic Processes - Papoulis
CD   Elements of Queueing Theory with Applications - Saaty
CE   Stochastic Processes - Doob
CF   Sample Survey Methods and Theory Vol 1 Methods and Applications Hansen, Hurwitz
CG   Advanced Statistical Methods in Biometric Research - C Radhakrishna Rao
CH   Introduction to the Mathematics of Statistics - Robert W. Burgess

Figure 6.6

Rank - Frequency of Reference
Distribution
Statistical Index Sample

Table 6.4

ABSTRACT ENTRIES FOR THE
AMALGAMATED STATISTICS INDEX SAMPLE

Normal distribution	Fisher, R. A.
Binomial distribution	Student
Poisson distribution	Pearson, E. S.
Degrees of freedom	Kendall, M. G.
Conditional probability	Bartlett, M. S.
Standard deviation	Cramer, H.
Analysis of variance	Neyman, J.

Distribution

Chi-square distribution

Central limit theorem

Least squares

Variance

Correlation coefficient

Median

Cauchy distribution

Covariance

Independence

Random variable

Exponential distribution

Gamma distribution

Moments

Bivariate normal distribution

Multinomial distribution

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as it does an immediate and objective overview of the
important subjects in statistics as well as the important
contributors. It plays the same role relative to that
portion of the field of statistics represented in the
monograph literature that the abstract entries for the
books described in Chapter 5 played; and it increases
the degree of information compression as well.

Figure 6.7 shows one page from the uncorrected form of the
amalgamated Statistics Index Sample described above.
This page has been selected to include the entry "log
normal" and those related to it. Observe that six books
(coded P, S, AD, BL, BU, CD) contain references to the
log normal distribution; since this represents only 7.5%
of the books in the Statistics Index Sample, the unsophis-
ticated inquirer will realize a very significant saving
in search time with a reasonable degree of assurance that
most of the significant references will either be covered
directly within these six books, or more comprehensive
treatments will be noted in their bibliographies.

Figure 6.7

Sample Page from the Amalgamated Statistics Index

```
AP LOCAL EXPLORATION,503
E  LOCAL MAXIMIN TEST,329,342
E  LOCAL OPTIMUM PROPERTIES OF TESTS,114,159,329,342,346
BG LOCAL PROBABILITY,54
Z  LOCALLY COMPACT SPACES,118,121,242
E  LOCALLY MAXIMIN TEST,329
E  LOCALLY MOST POWERFUL (LMP) TEST,159,342
CG LOCALLY MOST POWERFUL UNBIASED TESTS,280
BR LOCATING AVERAGES,252
BU LOCATION ERRORS,301,308,309,310,312,313,314
BT LOCATION INDICES, SEE ALSO BEST VALUE
BT LOCATION INDICES, SEE ALSO MEAN
BT LOCATION INDICES, SEE ALSO MEDIAN
BT LOCATION INDICES, SEE ALSO MODE
BT LOCATION INDICES, SEE ALSO MOST PROBABLE VALUE
BT LOCATION INDICES,76
AU LOCATION MEASURES,15
AM LOCATION OF EXPERIMENTS,211
AP LOCATION OF STATIONARY POINT,503-4
E  LOCATION PARAMETER FAMILY OF DISTRIBUTIONS IS STOCHASTICALLY INCREASING,73
AT LOCATION PARAMETER,144,179
BW LOCATION PARAMETER,62
Z  LOCATION PARAMETERS,44,134
AN LOCATION TEST CRITICAL VALUES,499, TABLE 18.4
AN LOCATION TEST DISTRIBUTIONS,499, TABLE 18.4
P  LOCATION,76
BP LOCKE,500
S  LOG NORMAL DISTRIBUTION,132
BU LOG NORMAL DISTRIBUTION,62,66
CD LOG NORMAL DISTRIBUTION,65
CD LOG NORMAL DISTRIBUTION,65
S  LOG NORMAL DISTRIBUTIONS,132
AT LOG-FACTOR,87,133
AT LOG-LIKELIHOOD,87
AZ LOG-LIKELOHOOD,4,24,38,46,64
AU LOG-NORMAL DISTRIBUTION,115,117
AD LOG-NORMAL DISTRIBUTION,115,117
P  LOG-NORMAL DISTRIBUTION,118
BL LOG-NORMAL DISTRIBUTION,95
AT LOG-ODDS,87
Z  LOGARITHM OF COMPLEX NUMBERS,532
AU LOGARITHM TABLE,204-205
AU LOGARITHM,190,191
BM LOGARITHMIC CHARTS DOUBLE LOGARITHMIC PAPER,17,18
BM LOGARITHMIC CHARTS SEMI-LOGARITHMIC PAPER,16,17
N  LOGARITHMIC DISTRIBUTION,291
Z  LOGARITHMIC DISTRIBUTION,62
AU LOGARITHMIC DISTRIBUTION,69
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BX LOGARITHMIC FORM SPECIAL REMARKS,195,198,201
BU LOGARITHMIC NORMAL DISTRIBUTION, SEE LOG NORMAL DISTRIBUTION
L  LOGARITHMIC PROBABILITY PAPER,91
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Appendix I

Abstract Index Entries:

A Uniform Sample from the

Fondren Index Sample

Marston, William Moulton   BF181.M3 1931
Integrative Psychology

Sum= 1432 / 29.54 = 48

23 Marston, W.M.   4 Angell, J.R.
                                    4 Archtypes
17 Freud, Sigmund   4 Cell body
                                    4 Compliance, motives
15 Watson, J.B.   4 Eng, H.
                                    4 Hering, E.
13 Cannon, W.B.   4 James-Lange, Theory of Motion
                                    4 Law of integrative sequence
11 Adler, Alfred   4 Origination response
                                    4 Passion motives
10 Desire   4 Submission
10 Jung, Carl   4 Trolaut, L.T.
10 Libido   4 Unit responses, compound
10 Woodsworth, R.S.   4 Washburn, M.F.
                                    4 Yerkes, R.M.
 9 Compliance
 9 Passion

 8 Allport, F.H.
 3 Behaviourism
 8 James, WM.
 8 MacDougall, Wm

 7 Herrick, C.J.
 7 Satisfaction
 7 Sherrington, C.S.

 6 Captivation
 6 Dominance
 6 Psychoanalysts

 5 Carlson, A.J.
 5 Erotic drive
 5 Inducement
 5 Passion response
 5 Visual discrimination, substances, hypothetical

McLean, Archibald   50
The History of the Foreign Missionary Society   BV2532.M3 1921

Sum= 498 / 29.54 = 16.86

7 Fallen, The

6 Moore, W.T.

4 Nurses being trained

3 Bilaspur
3 Johnson, Miss Kate V.
3 Loos, C.L.
3 Moore, W.T., Quoted
3 Rijnhardt, Dr. Susie C.

Coolidge, Archibald Cary   75
Ten Years of War and Peace   D443.C6 1927

Sum= 415 / 29.54 = 14

31 Great Britain mentioned

20 France, mentioned
20 Poland

19 League of Nations, mentioned

18 Versailles, Treaty of

16 Wilson, Woodrow

15 Hungary

13 Algeria
13 Hughes, Charles E., Secretary of State

11 Germany, mentioned
11 Harding, Warren G.
11 Morocco

10 China
10 France, estrangement between and Great Britain
10 Japan, mentioned
10 Rumania

Sackville-West, Victoria Mary   100
Knole and the Sackvilles   DA690.K7 1922

Sum= 307 / 29.54 = 10

7 Sackville, Lady Margaret(afterwards Countess of Thanet),
  mentioned in Lady Anne Clifford's diary

4 Pepys, Samuel, quoted
4 Walpole, Horace, quoted on Knole

3 Devonshire, Duchess of, his (i.e. 3rd Duke of Dorset) letter to
   her
3 Dryden, John, his debt to 6th Earl of Dorset
3 Gorboduc
3 Macaulay, quoted
3 Sackville, Charles, 6th Earl of Dorset, songs quoted
3 Sackville, Lord George, quoted
3 Wraxall, Sir Nathaniel, quoted

Sherrard, Philip
Byzantium

Sum= 1643 / 29.54^2 = 1.88

10 Churches: in Constantinople
10 Frescoes

Institute of Culture
The Cultural Heritage of India

Sum= 4906 / 29.54^2 = 5.6

51 Krsna (śrī)

46 Siva

41 "Bhagavad- Gītā"

37 Visnu

33 Brahman

32 Guru(s)

Saveth, Edward, ed.
Understanding the American Past

1958 / $29.54^2 = 2.24$

28 Beard, Charles A. and Mary

20 Jefferson, Thomas

17 Turner, Frederick Jackson

Link, Arthur S. 200

E741.L55 1963

American Epoch: A History of the United States Since the 1890's

Sum= 7016 / 29.54^2 = 8

14 Prices: agricultural

12 Foreign relations: Anglo-American

11 Federal income tax: individual
11 Tax: individual income

10 Farmers, income of
10 Legislation:agricultural

 9 Agriculture, legislation for
 9 Railroads: rates of

Bolton, Herbert Eugene
Anza's California Expedition

Sum= 648 / 29.54 = 22

132 Mass

111 Anza, Juan Bautista

60 Garces, Fray Francisco

46 Monterrey

44 San Gabriel Mission

27 San Diego mentioned.

26 Colorado River
26 Ribera (Rivera) Fernando de.
26 Sierra Madre de California

23 Apaches

21 Palma, Salvador
21 Sierra Nevada

20 San Miguel de Horcasitas

19 Eixarch (Eyxarch) Fray Thomás

18 San Francisco, harbor and settlement

17 Mexico

16 Spaniards

15 Christian Indians
15 Fages, Pedro
15 Gila River

14 Pablo (Captain Feo) Yuma Chief

13 Crespi, Fray Juan
13 Rio de San Francisco (San Joachin)

De Garmo, Ernest Paul
Engineering Economy

Sum= 650 / 29.54 = 22

5 Terborgh, Genge

4 Break even charts, examples of

3 Balance sheet, example of
3 Deferred-investment studies, examples of
3 Minimum cost point
3 Personnel factors, lighting
3 Rate of return, determination of
3 Selection, of design
3 Survivor curves, examples of

2 Accidents, effect of lighting on
2 Annuities whose present value is
2 Borrowed capital, cost of
2 Bureau of Internal Revenue relation to depreciation
2 Capital gains, and losses
2 Capital gains and losses, carry-over of
2 Capitalized cost, example of application
2 Costs, accuracy of estimates of
2 Costs, labor
2 Depreciation, sum-of-the-years'-digits
2 Hoover Dam
2 Income and expense statements, example of
2 Income taxes in public utility studies
2 Increment costs
2 Labor, turnover of
2 Life, economic
2 Life, useful
2 MAPI replacement formulas, forms for use in
2 Material, selection of
2 Multiple-purpose works, evaluation of benefits from
2 Overhead expense bases for distribution of
2 Plant location, economy studies of
2 Power factor, effect on utility rates
2 Rate schedules, block demand
2 Rautenstrauch, Walter
2 Risk, factors affecting
2 Selection of methods or processes
2 Self liquidating projects, relation of taxes to
2 Self liquidating projects, repayment of capital in
2 Wage payment, piece work

Chorafas, Dimitris N.   275
Operations Research for Industrial Management   HD20.C554 1958

Sum= 141 / 29.54 = 5

17 Charts on simulated business results

11 Computers usage
11 Simulation

10 Allocation

9 Managerial decisions

Smart, William

The Return to Protection

S= 201 / 29.54 = 7

20 Chamberlain, J.
20 Germany

19 Board of Trade

14 France

10 Giffen, Sir Robert
10 Shipping

9 America
9 America and protection
9 Canada

Cole, George Howard Douglas
Social Theory

Sum= 382 / 29.54 = 13

22 Trade Unions

15 "State, The"

14 Associations

12 Churches
12 Functional Equity, Court of, organisation

10 Law
10 Rousseau

9 Sovereignty

8 Function in relation to individual, perversion of
8 Marxism
8 Will, as a basis of Society

7 Middle Ages
7 Parliament

Utechin, Sergei
Russian Political Thought

Sum= 409 / 29.54 = 14

50 Economy

45 Classes, social

43 Law

39 Germany

34 Individualism
34 Monarchy

33 Emigration
33 France
33 Peasantry

32 Intelligentsia

31 Education

30 Property
30 Terror

29 Christianity
29 Culture
29 Equality
29 Moscow
29 Nationalism
29 Nobility
29 St. Petersburg

Cooper, Lane
Two Views of Education

Sum= 775 / 29.54 = 26

43 America

42 Milton

39 Plato

34 Shakespeare

27 Aristotle

26 Homer
26 Teacher (of English, etc.)

25 Greek, Study of

24 Middle Ages

22 Horace
22 Wordsworth

21 Bible

20 Latin, Study of

18 Cicero

17 Rewards of the Teacher

16 Odyssey
16 Rome
16 Virgil

15 Chaucer
15 Discipline
15 England
15 Socrates

14 Dante

13 Democracy
13 Greece
13 Rousseau

Morgan, Alexander
Education and Social Progress

400
LC191.M6 1916

Sum= 254 / 29.54 = 9

8 Children, diseases of
8 Education, practical
8 Inter-Departmental committee

7 Kindergartens
7 Practical education
7 Vocational education

6 Commission, Royal, on Poor Laws
6 Continuation education
6 Edinburgh, continuation schools
6 Education, continuation
6 Education, vocational
6 Education and health
6 Plato
6 Scotch Education Deptartment
6 Slums, children in

Tomkins, Calvin
The Bride and the Bachelors

Sum= 414 / 29.54 = 14

22 "Bride Stripped Bare By Her Bachelors, Even, The"
    (Duchamp)

15 Tudor, David

14 Cunningham, Merce

13 Rauschenberg, Robert

12 Klüver, Billy

11 Johns, Jasper

10 Duchamp, Marcel

8 Feldman, Morton
8 Thomson, Virgil
8 Tinguely, Jean

7 Arensberg,Walter C.
7 Breton, Andre
7 Cage, John
7 Cage, Mrs. John
7 Cowell, Henry
7 Dreier, Katherine
7 Kashevaroff, Xenia Andreevna
7 "Nu Descendant un Escalier" (Duchamp)
7 "Nude Descinding a Staircase" (Duchamp)
7 Schönberg, Arnold

Bobbe, Dorothic (De Bear)
Fanny Kemble

Sum= 384 / 29.54 = 13

42 Butler, Pierce

40 St. Leger, Harriet

36 Kemble, Charles

22 Butler, Sarah

19 Butler, Fanny
19 Siddons, Sarah

18 Covent Gargen Theatre
18 Lenox,Mass.

17 Kemble, Adelaide
17 Kemble, Mrs Charles
17 Sartoris, Mrs Edward
17 Slavery

15 Kemble, John Mitchell

Hoppe, Harry Reno   475
The Bad Quarto of Romeo and Juliet   PR2831.H6 1948

 Sum= 529 / 29.54 = 18

38 Greg, Walter W.

22 Chambers, (Sir) E.K.

18 Hart, Alfred
18 Mc Kerrow, R.B.

14 Burby, Cuthbert

13 Boswell, Eleanor
13 Greene, Robert "Orlando Furioso"

12 Arber, Edward
12 Recollections
12 Shakespeare, William,"The Merry Wives of Windsor"

11 "Orlando Furioso"

10 Anticipations
10 Chamberlain"s Company
10 Danter, John
10 Shakespeare, William "3 Henry VI"

9 "3 Henry VI"
9 "Merry Wives of Windsor, The "
9 Peele, George "The Old Wives" Tale"
9 Peele, George "Edward I"
9 Repetitions

Ryals, Clyde, de L
Theme and Symbol in Tennyson's Poems to 1850

Sum= 322 / 29.54 = 11

37 Keats, John

23 "In Memoriam"

22 William Wordsworth

20 "Two Voices, The"

17 "Palace of Art, The"

15 "Lotus Eaters, The" ·
15 "Ulyssus"

13 "Mariana"
13 "Recollections of the Arabian Nights"

12 Hallam, Arthur Henry

Kock, Ernst Albin, ed.
Den Norsk-Islandska Skaldediktningen

Sum= 626 / 29.54 = 21

2 Bjark: Bjarkamål, anon.
2 Danir. Danir, anon.
2 Finng: Finngálkn, anon.
2 Jómsvíkingar anon.
2 Karlevi: Karlevistenens drottkvädade vers, anon.
2 Oddm.: Oddmjor anon.
2 Rauðsk.: Rauðskeggr. anon.
2 Sveinn tjuguskegg, anon.
2 Svtjüg: Sveinn tjuguskegg, anon.
2 Tångbrand och Gudlev, dikt om, anon.
2 Vagn: Vagn Akason anon.
2 AEvidråpa (orvar-odds): ur Qrvar-odds saga

Ostrowski, Alexander
Vorlesungen Über Differential und Integralrechnung

Sum= 868 / 29.54 = 29

14 Cauchy, A.

9 Euler

6 Cantor, G.
6 Dirichlet
6 Gauss
6 Hardy, G.H.
6 Weierstrass

5 Abel
5 Ellipse
5 Hermite
5 Konvergenzkriterien für uneigentliche Integrale
5 Schwarz, H.A.

4 Bertand, J.
4 Bolzano
4 Cesàro
4 Hausdorff
4 Jensen, J.L.W.V.
4 Newton
4 Pringsheim, A.
4 Riemann, B.

3 Caratheòdory
3 Cauchy-Bolzanosches Konvergenzkriterium
3 Chaundy
3 Enveloppe
3 Fresnelsche Integrale
3 Hadamard
3 Inhalt
3 Konvergenzkriterium für unendliche Cauchy -Bolzanoshes
3 Poisson
3 Stieltjes
3 Vergluchskriterium für unendliche -uneigentliche Integrale
3 Zusammenhängend

Soule, Byron Avery
Library Guide for the Chemist

Sum= 1833 / 29.54 = 28

5 Gregory

4 Böttger, Wm.
4 Furman, N.H.
4 Water, analysis of

3 Biography, German
3 Browne, C.A.
3 Classen
3 Daniels, F.
3 Dyes, patents on
3 Ferro-alloys, analysis
3 Findlay, A.
3 Glasstone, S.
3 Hahn, D.
3 Hall, W.T.
3 Houben, J.
3 Indexes, patent
3 Koltoff, I.M.
3 Martin, G.
3 Meyer, R.J.
3 Mnemonics
3 Nomenclature, organic
3 Organo-metallic Compounds
3 Ostwald, Wm.
3 Patents, dye
3 Rossman, J.
3 Steel, analysis of
3 Sugar, analysis
3 Thorpe, Edw.
3 Weiser, H.B.
3 Worden, E.C.

Varley, Ernest Reginald   600
Sillimanite   QE391.S5 V3 1965

Sum= 729 / 29.54 = 25

11 Reserves, India

10 Andalusite: U.S.A.

8 Kenya
8 Reserves, U.S.A.

7 Assam, India
7 United States

6 Florida, U.S.A.
6 Georgia, U.S.A.

5 Beneficiation, U.S.A.
5 Bihar, India
5 Brazil
5 California, U.S.A.
5 Dumortierite, U.S.A.
5 Mysore, India
5 Nyasaland
5 South Africa, Republic of
5 Topaz: U.S.A.

4 Aluminium industry
4 Andalusite: U.S.S.R.
4 Andhra Pradesh, India
4 Baker Mountain, Virginia
4 Density
4 Graves Mountain, Georgia
4 Henry Knob, S. Carolina
4 India
4 Kerala, India
4 Kyanite density
4 Lapsa Buru, Bihar
4 Madhya Pradesh, India
4 Maharashtra, India
4 Nevada, U.S.A.
4 New South Wales, Australia
4 Orissa, India
4 Reserves, U.S.S.R.
4 Sillimanite minerals:density
4 South Carolina U.S.A.
4 Transvaal, South Africa
4 United States, National Stockpile Purchase Specification

Davis, David Edward
Principles in Mammalogy

625
QL703.D3 1963

Sum= 836 / 29.54 = 28

11 Carnivores

10 Bat(s)
10 Insectivores
10 Woodchucks

9 Marsupials
9 Mutation
9 Teeth

8 Monotremes
8 Whale(s)

7 Herbivores
7 Opossums
7 European rabbit(s)
7 Raccoons

6 Dispersal
6 Maintenance
6 Predators
6 Primates
6 Shrew(s)
6 Vole, meadow

5 Body size, temperature
5 Camels
5 Competition
5 Corpora lutea
5 Elephants
5 Feedback
5 Food
5 Fossils
5 Migration
5 Mole
5 Muskrats
5 Nearctic region
5 Omnivores
5 Oriental region
5 Sex ratio
5 Squirrel(s), ground
5 Temperature

Ewerhardt, Frank Henry
Therapeutic Exercise

Sum= 338 / 29.54 = 11

8 Muscle contraction
8 Paralysis
8 Posture

7 Spastic paralysis, exercise in

6 Flat foot
6 Muscle function volitional tests
6 Poliomyelitis treatment
6 Re-education

5 Hemiplegia
5 Lordosis
5 Poliomyelitis testing by topographical observations
5 Poliomyelitis treatment during acute stage
5 Re-education of upper extremity
5 Scoliosis
5 Upper extremity re-education
5 Zero position

Underhill, Charles Reginald
Electrons at Work

Sum= 3827 / 29.54^2 = 4

9 Tube, gaseous-discharge

7 Hertz
7 Light, ultra-violet
7 Maxwell
7 Reaction, Reactors
7 Valence electrons

Bibliography of Medieval Drama
Stratman, Carl Joseph

Sum= 2797 / 29.54^2 = 3.2

44 Passion

37 Comedy

32 Latin

31 Staging

APPENDIX II

PAGE REFERENCE DISTRIBUTIONS

FROM

THE FONDREN INDEX SAMPLE

F864.b68 V4 1930

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QD9.S71 Ref. 1938

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TK153 U5 1933

10^4

10^3

10^2

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Z5782.A258 Ref. 1954

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BF181.M3 1931

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NUMBER OF PAGE REFERENCES

E178.6.S3 1965

NUMBER OF INDEX ENTRIES

NUMBER OF PAGE REFERENCES

DF521.S4 1966

NUMBER OF INDEX ENTRIES

NUMBER OF PAGE REFERENCES

DS423.C85 V4 1953-58

NUMBER OF INDEX ENTRIES

NUMBER OF PAGE REFERENCES

NUMBER OF INDEX ENTRIES

JA84.R9 U8 1964

10^3

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HM66.C7 1920

10^3

10^2

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10^2

NUMBER OF PAGE REFERENCES

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10^3 10^2 10   1

HD20.C554 1958

NUMBER OF INDEX ENTRIES

NUMBER OF PAGE REFERENCES

10^3 10^2 10   1

HF2046.562 1904

NUMBER OF INDEX ENTRIES

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BV2532.M3 1921

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D443.C6 1927

NUMBER OF PAGE REFERENCES

10^3 10^2 10 1

LB875.C7 1922

NUMBER OF INDEX ENTRIES

NUMBER OF PAGE REFERENCES

10^3 10^2 10 1

LC191.M6 1916

NUMBER OF INDEX ENTRIES

211

NUMBER OF PAGE REFERENCES

DA690.K7 1922

NUMBER OF INDEX ENTRIES

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HB199.W595 1960

NUMBER OF INDEX ENTRIES

NUMBER OF PAGE REFERENCES

QE391.S5 V3 1965

NUMBER OF INDEX ENTRIES

NUMBER OF PAGE REFERENCES

QL703.D3 1963

NUMBER OF INDEX ENTRIES

NUMBER OF PAGE REFERENCES

PT7244.K57 1946-49

NUMBER OF INDEX ENTRIES

NUMBER OF PAGE REFERENCES

QA303.088 V3 1945-54

NUMBER OF INDEX ENTRIES

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PR5588.R9 1964

NUMBER OF PAGE REFERENCES

NUMBER OF INDEX ENTRIES

PR2831.H6 1948

NUMBER OF PAGE REFERENCES

NUMBER OF INDEX ENTRIES

RM721.E8 1947

10^3

10^2

10

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1

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AMALGAM^TED ALGORITHMIC INDEX TO

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